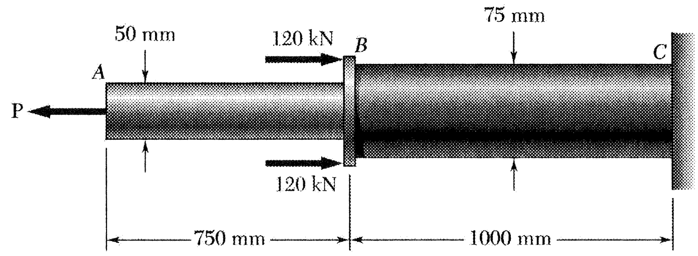
Chapter 1

Problem 1.1

1.1 Two solid cylindrical rods *AB* and *BC* are welded together at B and loaded as shown. Determine the magnitude of the force P for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.



$$A_{AB} = \frac{\pi}{4}(50)^2 = 1963.5\ mm^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{1963.5} = 509.3\times10^{-6}P$$

$$A_{BC} = \frac{\pi}{4}(75)^2 = 4417.9\ mm^2$$

$$\sigma_{BC} = \frac{(2)(120) - P}{A_{AB}} = \frac{240 - P}{4417.9} = 0.0543 - 226.4\times10^{-6}P$$

Equating σ_{AB} to $2\,\sigma_{BC}$

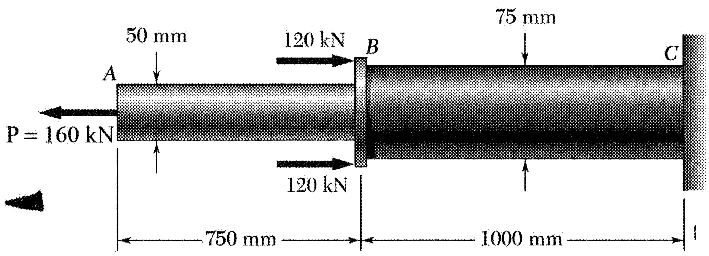
$$509.3\times10^{-6}P = 2(0.0543 - 226.4\times10^{-6}P)$$

$$P = 112.9\ kN$$

Problem 1.2

1.2 In Prob. 1.1, knowing that $P = 160$ kN, determine the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

1.1 Two solid cylindrical rods *AB* and *BC* are welded together at B and loaded as shown. Determine the magnitude of the force P for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.



(a) Rod AB.

$P = 160\ kN$ (tension)

$$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi(50)^2}{4} = 1963.5\ mm^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{160\times10^3}{1963.5} = 81.5\ MPa$$

(b) Rod BC.

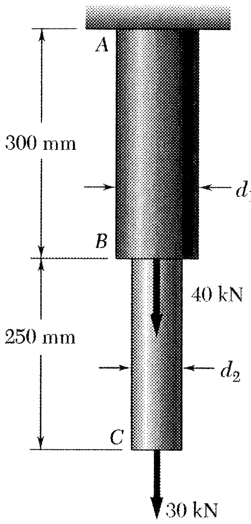
$$F = 160 - (2)(120) = -80\ kN$$ i.e. 80 kN compression.

$$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi(75)^2}{4} = 4417.9\ mm^2$$

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-80\times10^3}{4417.9}$$

$$\sigma_{BC} = -18.1\ MPa$$

Problem 1.3



1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod *AB* and 150 MPa in rod *BC*, determine the smallest allowable values of d_1 and d_2.

Rod AB.

$$P = 40 + 30 = 70 \text{ kN} = 70\times10^3 \text{ N}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\frac{\pi}{4}d_1^2} = \frac{4P}{\pi d_1^2}$$

$$d_1 = \sqrt{\frac{4P}{\pi\sigma_{AB}}} = \sqrt{\frac{(4)(70\times10^3)}{\pi(175\times10^6)}} = 22.6\times10^{-3} \text{ m}$$

$$d_1 = 22.6 \text{ mm}$$

Rod BC.

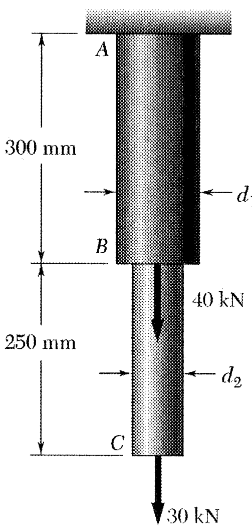
$$P = 30 \text{ kN} = 30\times10^3 \text{ N}$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\frac{\pi}{4}d_2^2} = \frac{4P}{\pi d_2^2}$$

$$d_2 = \sqrt{\frac{4P}{\pi\sigma_{BC}}} = \sqrt{\frac{(4)(30\times10^3)}{\pi(150\times10^6)}} = 15.96\times10^{-3} \text{ m}$$

$$d_2 = 15.96 \text{ mm}$$

Problem 1.4



1.4 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that d_1 = 50 mm and d_2 = 30 mm, find average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

(a) Rod AB.

$$P = 40 + 30 = 70 \text{ kN} = 70\times10^3 \text{ N}$$

$$A = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(50)^2 = 1.9635\times10^3 \text{ mm}^2 = 1.9635\times10^{-3} \text{ m}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{70\times10^3}{1.9635\times10^{-3}} = 35.7\times10^6 \text{ Pa}$$

$$\sigma_{AB} = 35.7 \text{ MPa}$$

(b) Rod BC.

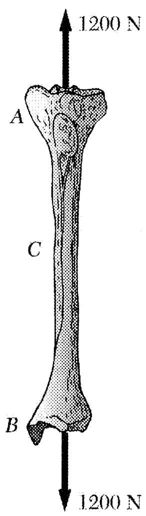
$$P = 30 \text{ kN} = 30\times10^3 \text{ N}$$

$$A = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times10^{-6} \text{ m}^2$$

$$\sigma_{BC} = \frac{P}{A} = \frac{30\times10^3}{706.86\times10^{-6}} = 42.4\times10^6 \text{ Pa}$$

$$\sigma_{BC} = 42.4 \text{ MPa}$$

Problem 1.5



1.5 A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \frac{\pi}{4}(d_1^2 - d_2^2)$

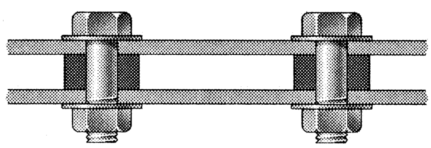
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25\times10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80\times10^6)}$$

$$= 222.9\times10^{-6}\ \text{m}^2$$

$$d_2 = 14.93\times10^{-3}\ \text{m} \qquad d_2 = 14.93\ \text{mm}$$

Problem 1.6



1.6 Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time the spacer pushes that plate upward with a compressive force P_s. In order to maintain equilibrium

$$P_b = P_s$$

For the bolt, $\sigma_b = \frac{F_b}{A_b} = \frac{4P_b}{\pi d_b^2}$ or $P_b = \frac{\pi}{4}\sigma_b d_b^2$

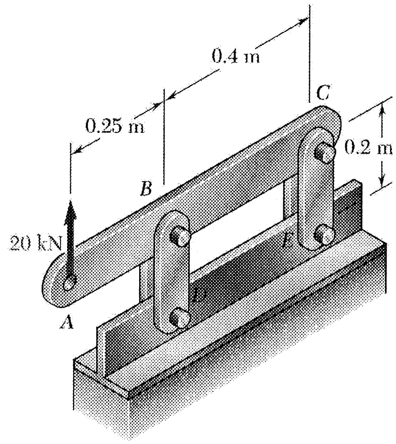
For the spacer, $\sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

Equating P_b and P_s,

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

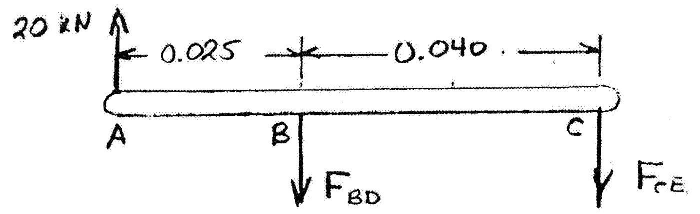
$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)}\, d_b = \sqrt{1 + \frac{200}{130}}\,(16) \qquad d_s = 25.2\ \text{mm}$$

Problem 1.7



1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

Use bar ABC as a free body.



$\Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20\times10^3) = 0$

$F_{BD} = 32.5\times10^3$ N $\qquad$ Link BD is in tension.

$\Sigma M_B = 0: \quad -(0.040)F_{CE} - (0.025)(20\times10^3) = 0$

$F_{CE} = -12.5\times10^3$ N $\qquad$ Link CE is in compression.

Net area of one link for tension $= (0.008)(0.036 - 0.016)$

$= 160\times10^{-6}$ m². For two parallel links, $A_{net} = 320\times10^{-6}$ m²

<u>Tensile stress in link BD.</u>

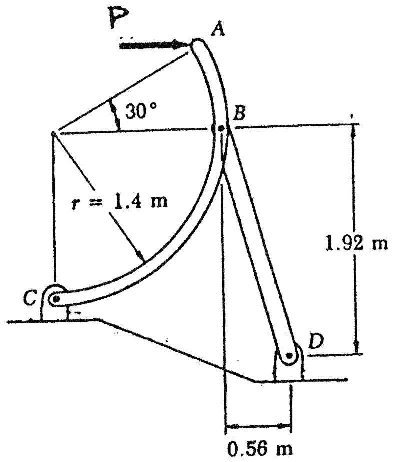
(a) $\quad \sigma_{BD} = \dfrac{F_{BD}}{A_{net}} = \dfrac{32.5\times10^3}{320\times10^{-6}} = 101.56\times10^6 \qquad \sigma_{BD} = 101.6$ MPa ◄

Area for one link in compression $= (0.008)(0.036)$

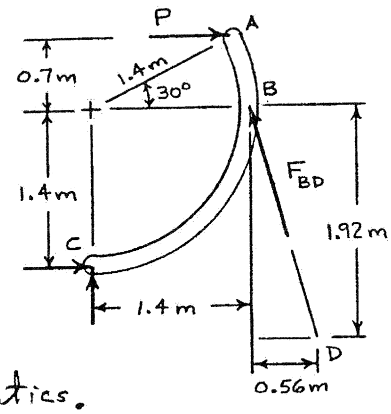
$= 288\times10^{-6}$ m². For two parallel links, $A = 576\times10^{-6}$ m²

(b) $\quad \sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5\times10^3}{576\times10^{-6}} = -21.70\times10^6 \qquad \sigma_{CE} = -21.7$ MPa ◄

Problem 1.8



1.8 Knowing that the central portion of the link *BD* has a uniform cross-sectional area of 800 mm^2, determine the magnitude of the load **P** for which the normal stress in that portion of *BD* is 50 MPa.



$$F_{BD} = \sigma A$$
$$= (50 \times 10^6)(800 \times 10^{-6})$$
$$= 40 \times 10^3 \text{ N}$$

$$BD = \sqrt{(0.56)^2 + (1.92)^2}$$
$$= 2.00 \text{ m}$$

Use Free Body AC for statics.

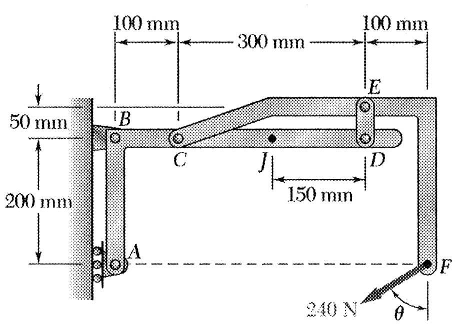
$+\circlearrowleft \Sigma M_C = 0:$

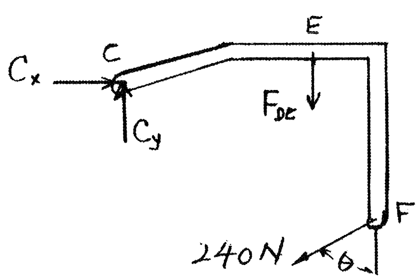
$$\frac{0.56}{2.00}(40 \times 10^3)(1.4) + \frac{1.92}{2.00}(40 \times 10^3)(1.4)$$
$$- P(0.7 + 1.4) = 0$$

$$P = 33.1 \times 10^3 \text{ N} \qquad P = 33.1 \text{ kN}$$

Problem 1.9

1.9 Knowing that link *DE* is 25 mm wide and 3 mm thick, determine the normal stress in the central portion of that link when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.





Use member CEF as a free body

$+\circlearrowleft \Sigma M_C = 0$

$$-0.3\, F_{DE} - (0.2)(240 \sin\theta) - (0.4)(240 \cos\theta) = 0$$

$$F_{DE} = -160 \sin\theta - 320 \cos\theta \text{ N}$$

$$A_{DE} = (0.025)(0.003) = 75 \times 10^{-6} \text{ m}^2$$

$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$$

(a) $\theta = 0$: $\quad F_{DE} = -320 \text{ N}$

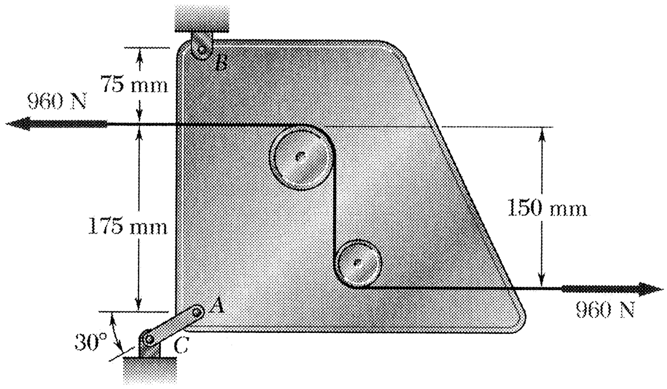
$$\sigma_{DE} = \frac{-320}{75 \times 10^{-6}} = -4.27 \text{ MPa}$$

(b) $\theta = 90°$: $\quad F_{DE} = -160 \text{ N}$

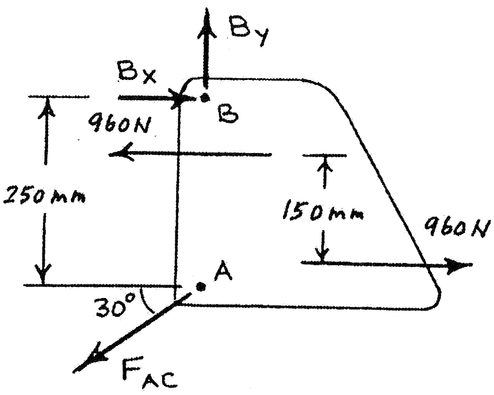
$$\sigma_{DE} = \frac{-160}{75 \times 10^{-6}} = -2.13 \text{ MPa}$$

Problem 1.10

1.10 Link *AC* has a uniform rectangular cross section 3 mm thick and 12 mm wide. Determine the normal stress in the central portion of the link.



Free Body Diagram of Plate



Note that the two 960-N forces form a couple of moment

$$(960\ N)(0.15 m) = 144\ N.m$$

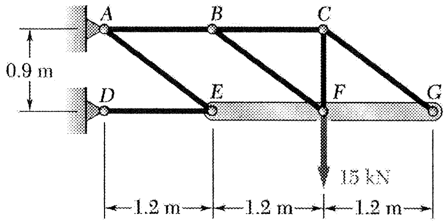
$$\circlearrowleft \Sigma M_B = 0: \quad 144\ N.m - (F_{AC}\cos 30^\circ)(0.25\ m) = 0$$

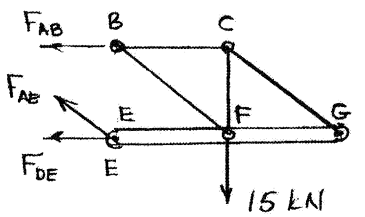
$$F_{AC} = 665.1\ N$$

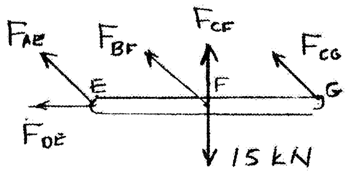
Area of link: $A_{AC} = (3\ mm)(12\ mm) = 36\ mm^2$

Stress $\sigma_{AC} = \dfrac{F_{AC}}{A_{AC}} = \dfrac{665.1}{36} = 18.475\ MPa \qquad \sigma_{AC} = 18.5\ MPa$

Problem 1.11







1.11 The rigid bar *EFG* is supported by the truss system shown. Knowing that the member *CG* is a solid circular rod of 18 mm diameter, determine the normal stress in *CG*.

Using portion EFGCB as a free body

$+\uparrow \Sigma F_y = 0: \frac{0.9}{1.5} F_{AB} - 15 = 0$

$F_{AE} = 25\ kN$

Using beam EFG as a free body

$\circlearrowleft M_F = 0: -(1.2)\frac{0.9}{1.2} F_{AE} + (1.2)\left(\frac{0.9}{1.2} F_{CG}\right) = 0$

$F_{CG} = F_{AE} = 25\ kN$

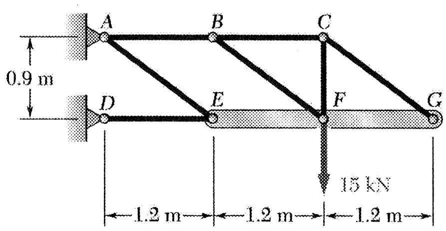
Cross sectional area of member CG

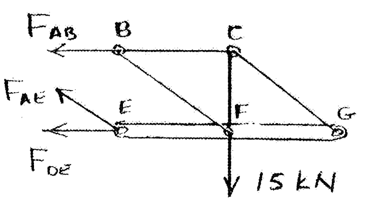
$A_{CG} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.018)^2 = 254.4 \times 10^{-6}\ m^2$

Normal stress in CG.

$\sigma_{CG} = \frac{F_{CG}}{A_{CG}} = \frac{25}{254.4 \times 10^{-6}} = 98.3\ MPa$ ◄

Problem 1.12





1.12 The rigid bar *EFG* is supported by the truss system shown. Determine the cross-sectional area of member *AE* for which the normal stress in the member is 105 MPa.

Using portion EFGCB as a free body

$+\uparrow \Sigma F_y = 0: \frac{0.9}{1.5} F_{AE} - 15 = 0$

$F_{AE} = 25\ kN$

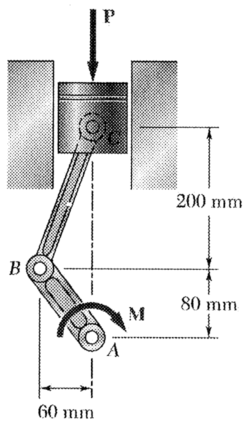
Stress in member AE $\qquad \sigma_{AE} = 105\ MPa$

$\sigma_{AE} = \frac{F_{AE}}{A_{AE}}$

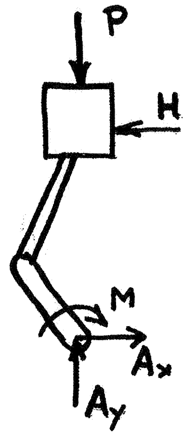
$A_{AE} = \frac{F_{AE}}{\sigma_{AE}} = \frac{25 \times 10^3}{105 \times 10^{-6}} = 238.1 \times 10^{-6}\ m^2$ ◄

Problem 1.13

1.13 A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm² uniform cross section.



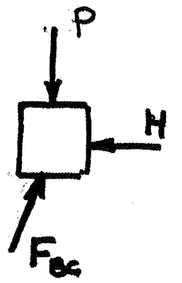
Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

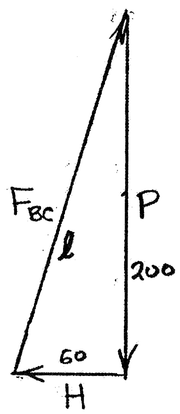


$$\circlearrowleft \Sigma M_A = 0:$$

$$(0.280\ \text{m})H - 1500\ \text{N}\cdot\text{m} = 0$$

$$H = 5.3571 \times 10^3\ \text{N}$$





Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BE} by proportions.

$$\ell = \sqrt{200^2 + 60^2} = 208.81\ \text{mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3\ \text{N}$$

(a) $P = 17.86\ \text{kN}$ ◄

$$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.643 \times 10^3\ \text{N}$$

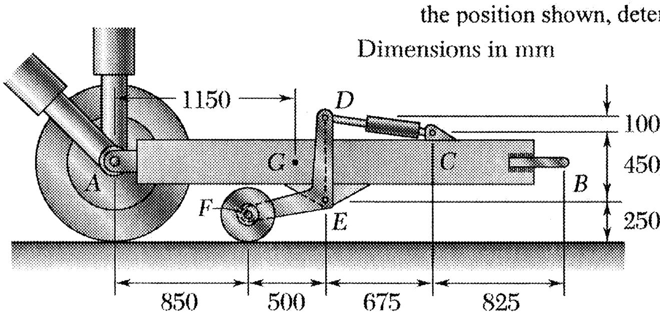
Rod BC is a compression member. Its area is $450\ \text{mm}^2 = 450 \times 10^{-6}\ \text{m}^2$

Stress, $\sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6\ \text{Pa}$

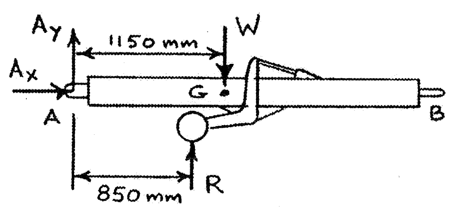
(b) $\sigma_{BC} = -41.4\ \text{MPa}$ ◄

Problem 1.14

1.14 An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.



FREE BODY – ENTIRE TOW BAR:



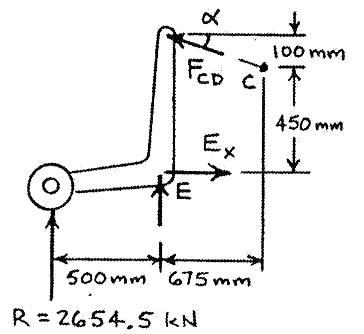
$W = (200\text{ kg})(9.81\text{ m/s}^2) = 1962.00\text{ N}$

$+\circlearrowleft \Sigma M_A = 0:$

$850R - 1150(1962.00\text{ N}) = 0$

$R = 2654.5\text{ N}$

FREE BODY – BOTH ARM & WHEEL UNITS:



$\tan\alpha = \dfrac{100}{675} \qquad \alpha = 8.4270°$

$+\circlearrowleft \Sigma M_E = 0:$

$(F_{CD}\cos\alpha)(550) - R(500) = 0$

$F_{CD} = \dfrac{500}{550\cos 8.4270°}(2654.5\text{ N})$

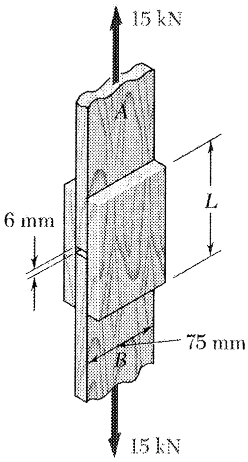
$= 2439.5\text{ N} \quad (\text{COMP.})$

$\sigma_{CD} = -\dfrac{F_{CD}}{A_{CD}} = -\dfrac{2439.5\text{ N}}{\pi(0.0125\text{ m})^2}$

$= -4.9697 \times 10^6\text{ Pa}$

$\sigma_{CD} = -4.97\text{ MPa}$ ◀

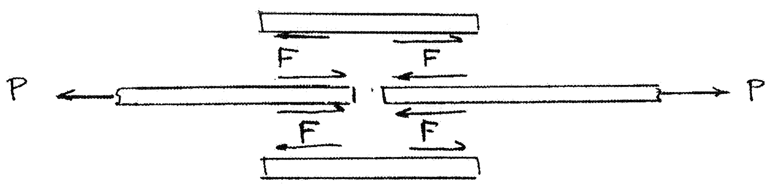
Problem 1.15



1.15 The wooden members A and B are to be joined by plywood splice plates that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be 6 mm, determine the smallest allowable length L if the average shearing stress in the glue is not to exceed 700 kPa.

There are four separate areas that are glued. Each of these areas transmits one half the the 15 kN load. Thus

$$F = \frac{1}{2}P = \frac{1}{2}(15) = 7.5 \text{ kN} = 7500 \text{ N}$$



Let l = length of one glued area and $w = 75 \text{ mm} = 0.075 \text{ m}$ be its width.

For each glued area, $A = lw$

Average shearing stress: $\tau = \frac{F}{A} = \frac{F}{lw}$

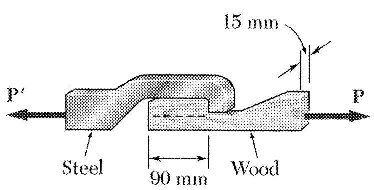
The allowable shearing stress is $\tau = 700 \times 10^3$ Pa

Solving for l, $l = \frac{F}{\tau w} = \frac{7500}{(700 \times 10^3)(0.075)} = 0.14286 \text{ m} = 142.85 \text{ mm}$

Total length L: $L = l + (\text{gap}) + l = 142.85 + 6 + 142.85$

$L = 292$ mm ◄

Problem 1.16



1.16 When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

Area being sheared

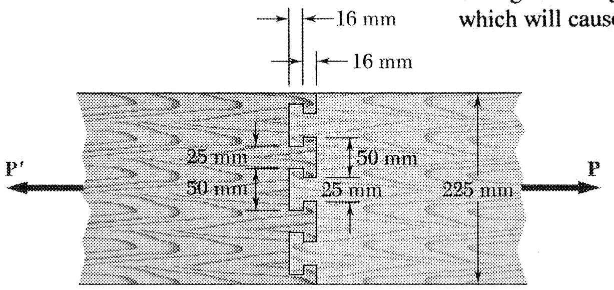
$A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{ m}^2$

Force $P = 8 \times 10^3$ N

Shearing stress $\tau = \frac{P}{A} = \frac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^3 \text{ Pa} = 5.93 \text{ MPa}$ ◄

Problem 1.17

1.17 Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load which will cause the joint to fail.



Six areas must be sheared off when the joint fails. Each of these areas has dimensions 16 mm × 12 mm, its area being

$$A = (16)(12) = 192 \text{ mm}^2 = 192 \times 10^{-6} \text{ m}^2$$

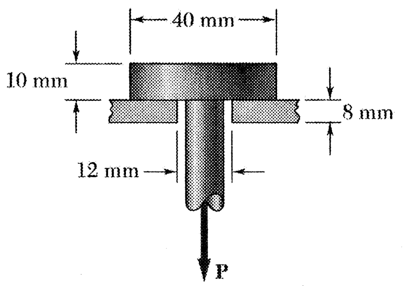
At failure the force F carried by each of areas is

$$F = \tau A = (8 \times 10^6)(192 \times 10^{-6}) = 1536 \text{ N} = 1.536 \text{ kN}$$

Since there are six failure areas $\quad P = 6F = (6)(1.536) = 9.22 \text{ kN}$ ◄

Problem 1.18

1.18 A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.



For the steel rod,

$$A_1 = \pi d_1 t_1 = (\pi)(0.012)(0.010)$$
$$= 376.99 \times 10^{-6} \text{ m}^2$$

$$\tau_1 = \frac{P}{A_1} \rightarrow P_1 = \tau_1 A_1$$

$$P_1 = (180 \times 10^6)(376.99 \times 10^{-6}) = 67.86 \times 10^3 \text{ N}$$

For the aluminum plate,

$$A_2 = \pi d_2 t_2 = (\pi)(0.040)(0.008) = 1.00531 \times 10^{-3} \text{ m}^2$$

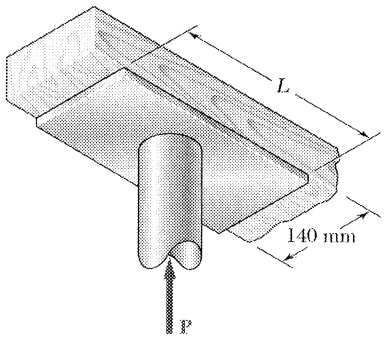
$$\tau_2 = \frac{P_2}{A_2} \Rightarrow P_2 = \tau_2 A_2$$

$$P_2 = (70 \times 10^6)(1.0053 \times 10^{-6}) = 70.372 \times 10^3 \text{ N}$$

The limiting value for the load P is the smaller of P_1 and P_2.

$P = 67.86 \times 10^3$ N $\qquad P = 67.9$ kN ◄

Problem 1.19



1.19 The axial force in the column supporting the timber beam shown is $P = 75$ kN. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 3.0 MPa.

SOLUTION

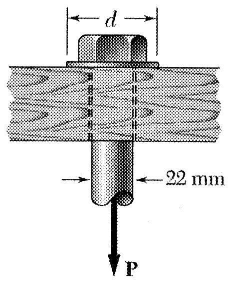
$$\sigma_b = \frac{P}{A} = \frac{P}{Lw}$$

Solving for L: $L = \dfrac{P}{\sigma_b w} = \dfrac{75 \times 10^3}{(3.0 \times 10^6)(0.140)}$

178.6×10^{-3} m

$L = 178.6$ mm

Problem 1.20



1.20 The load **P** applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.

Steel rod: $A = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6}\ \text{m}^2$

$\sigma = 35 \times 10^6$ Pa

$P = \sigma A = (35 \times 10^6)(380.13 \times 10^{-6})$

$= 13.305 \times 10^3$ N

Washer: $\sigma_b = 5 \times 10^6$ Pa

Required bearing area:

$$A_b = \frac{P}{\sigma_b} = \frac{13.305 \times 10^3}{5 \times 10^6} = 2.6609 \times 10^{-3}\ \text{m}^2$$

But, $A_b = \frac{\pi}{4}(d^2 - d_i^2)$

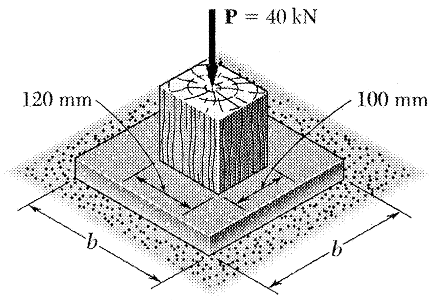
$d^2 = d_i^2 + \dfrac{4A_b}{\pi}$

$= (0.025)^2 + \dfrac{(4)(2.6609 \times 10^{-3})}{\pi}$

$= 4.013 \times 10^{-3}\ \text{m}^2$

$d = 63.3 \times 10^{-3}$ m $\qquad d = 63.3$ mm

Problem 1.21



1.21 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

(a) Bearing stress on concrete footing.

$$P = 40\ \text{kN} = 40 \times 10^3\ \text{N}$$

$$A = (100)(120) = 12 \times 10^3\ \text{mm}^2 = 12 \times 10^{-3}\ \text{m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.33 \times 10^6\ \text{Pa}$$

3.33 MPa

(b) Footing area.

$$P = 40 \times 10^3\ \text{N} \qquad \sigma = 145\ \text{kPa} = 45 \times 10^3\ \text{Pa}$$

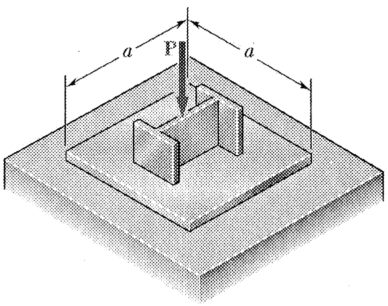
$$\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586\ \text{m}^2$$

Since the area is square, $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525\ \text{m}$$

$b = 525$ mm

Problem 1.22



1.22 An axial load **P** is supported by a short W200 × 59 column of cross-sectional area A = 7560 mm^2 and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 200 MPa and that the bearing stress on the concrete foundation must not exceed 20 MPa, determine the side *a* of the plate that will provide the most economical and safe design.

For the column $\sigma = \frac{P}{A}$

or $P = \sigma A = (200 \times 10^6)(7560 \times 10^{-6}) = 1512\ \text{kN}$

For the a × a plate, $\sigma = 20\ \text{MPa}$

$$A = \frac{P}{\sigma} = \frac{1.512}{20} = 0.0756\ \text{m}^2$$

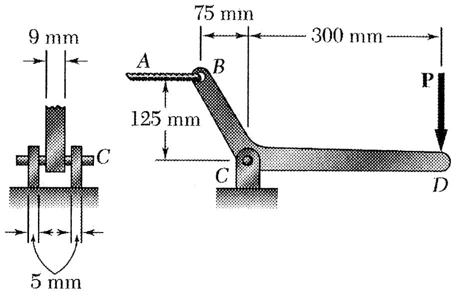
Since the plate is square $A = a^2$

$$a = \sqrt{A} = \sqrt{0.0756} = 0.275\ \text{m}$$

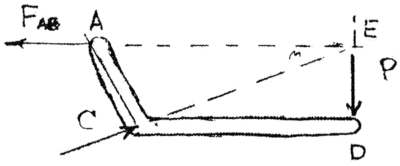
$= 275$ mm

Problem 1.23

1.23 A 6-mm-diameter pin is used at connection C of the pedal shown. Knowing that $P = 500$ N, determine (*a*) the average shearing stress in the pin, (*b*) the nominal bearing stress in the pedal at C, (*c*) the nominal bearing stress in each support bracket at C.



Draw free body diagram of ACD. Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm.

$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325} C - P = 0 \qquad C = 2.6P = (2.6)(500) = 1300$ N

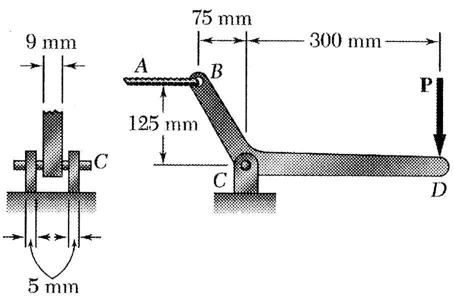
(a) $\tau_{pin} = \frac{\frac{1}{2}C}{A_{pin}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} = \frac{2C}{\pi d^2} = \frac{(2)(1300)}{\pi(6\times10^{-3})^2} = 23.0\times10^6$ Pa $\qquad \tau_{pin} = 23.0$ MPa ◄

(b) $\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1300}{(6\times10^{-3})(9\times10^{-3})} = 24.1\times10^6$ Pa $\qquad \sigma_b = 24.1$ MPa ◄

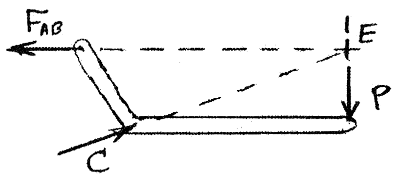
(c) $\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1300}{(2)(6\times10^{-3})(5\times10^{-3})} = 21.7\times10^6$ Pa $\qquad \sigma_b = 21.7$ MPa ◄

Problem 1.24

1.24 Knowing that a force **P** of magnitude 750 N is applied to the pedal shown, determine (*a*) the diameter of the pin at C for which the average shearing stress in the pin is 40 MPa, (*b*) the corresponding bearing stress in the pedal at C, (*c*) the corresponding bearing stress in the each support bracket at C.



Draw free body diagram of ACD. Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm

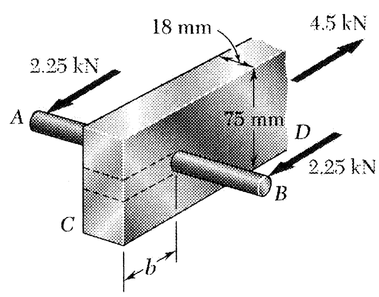
$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325} C - P = 0 \qquad C = 2.6P = (2.6)(750) = 1950$ N

(a) $\tau_{pin} = \frac{\frac{1}{2}C}{A_{pin}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} \qquad d = \sqrt{\frac{2C}{\pi \tau_{pin}}} = \sqrt{\frac{(2)(1950)}{\pi(40\times10^6)}} = 5.57\times10^{-3}$ m $\qquad d = 5.57$ mm ◄

(b) $\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1950}{(5.57\times10^{-3})(9\times10^{-3})} = 38.9\times10^6$ Pa $\qquad \sigma_b = 38.9$ MPa ◄

(c) $\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1950}{(2)(5.57\times10^{-3})(5\times10^{-3})} = 35.0\times10^6$ Pa $\qquad \sigma_b = 35.0$ MPa ◄

Problem 1.25



1.25 A 12-mm-diameter steel rod *AB* is fitted to a round hole near end *C* of the wooden member *CD*. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance *b* for which the average shearing stress is 620 kPa on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

(a) Maximum average normal stress in the wood.

$$A_{net} = (75-12)(18) = 1.134 \times 10^3 \text{ mm}^2 = 1.134 \times 10^{-3} \text{ m}^2$$

$$P = 4.50 \text{ kN} = 4.50 \times 10^3 \text{ N}$$

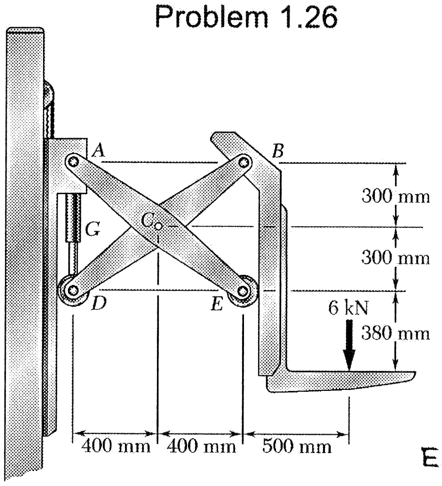
$$\sigma = \frac{P}{A_{net}} = \frac{4.50 \times 10^3}{1.134 \times 10^{-3}} = 3.97 \times 10^6 \text{ Pa} \qquad 3.97 \text{ MPa}$$

(b) $$\tau = \frac{P}{A} = \frac{P}{2bt} \qquad b = \frac{P}{2t\tau} = \frac{4.50 \times 10^3}{(2)(18 \times 10^{-3})(620 \times 10^3)} = 202 \times 10^{-3} \text{ m}$$

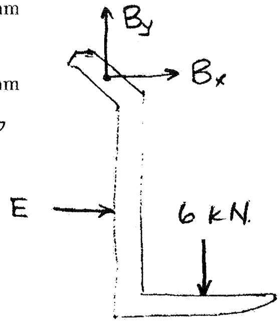
$$b = 202 \text{ mm}$$

(c) $$\sigma_b = \frac{P}{dt} = \frac{4.50 \times 10^3}{(12 \times 10^{-3})(18 \times 10^{-3})} = 20.8 \times 10^6 \text{ Pa} \qquad 20.8 \text{ MPa}$$

Problem 1.26



1.26 Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 6 kN. Knowing that the thickness of member *BD* is 16 mm, determine (*a*) the average shearing stress in the 12-mm-diameter pin at *B*, (*b*) the bearing stress at *B* in member *BD*.



Use one fork as a free body.

$+\circlearrowleft \Sigma M_B = 0:$

$0.6E - (0.5)(6) = 0$

$E = 5 \text{ kN} \rightarrow$

$\xrightarrow{+} \Sigma F_x = 0$

$E + B_x = 0 \qquad B_x = -E$

$B_x = 5 \text{ kN} \leftarrow$

$+\uparrow \Sigma F_y = 0: \quad B_y - 6 = 0 \qquad B_y = 6 \text{ kN.}$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{5^2 + 6^2} = 7.81 \text{ kN}$$

(a) Shearing stress in pin at B.

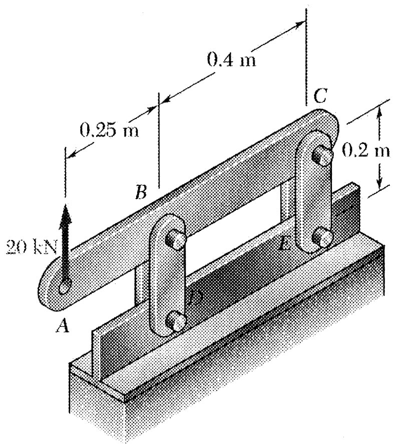
$$A_{pin} = \frac{\pi}{4} d_{pin}^2 = \frac{\pi}{4}(0.012)^2 = 113.1 \times 10^{-6} \text{ m}^2$$

$$\tau = \frac{B}{A_{pin}} = \frac{7.81 \times 10^3}{113.1 \times 10^{-6}} = 69 \text{ MPa}$$

(b) Bearing stress at B.

$$\sigma = \frac{B}{dt} = \frac{7.8 \times 10^3}{(0.012)(0.016)} = 40.6 \text{ MPa}$$

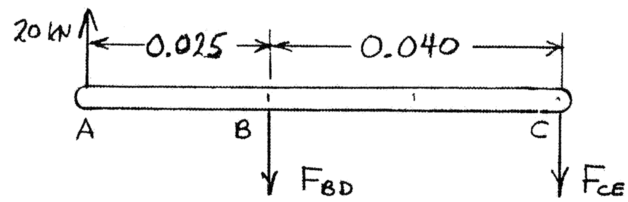
Problem 1.27



1.27 For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

Use bar ABC as a free body.



$$\circlearrowleft \Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(a) <u>Shear pin at B.</u> $\quad \tau = \dfrac{F_{BD}}{2A}$ for double shear,

where $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \qquad \tau = 80.8 \text{ MPa} \blacktriangleleft$$

(b) <u>Bearing: link BD.</u> $\quad A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \qquad \sigma_b = 127.0 \text{ MPa} \blacktriangleleft$$

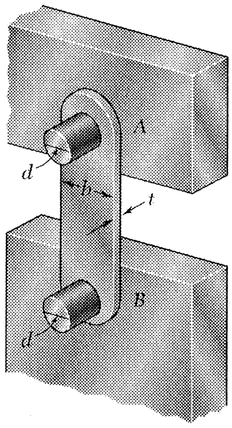
(c) <u>Bearing in ABC at B.</u>

$$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \qquad \sigma_b = 203 \text{ MPa} \blacktriangleleft$$

Problem1.28

1.28 Link AB, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -140 MPa, and that the average shearing stress in each of the two pins is 80 MPa, determine (*a*) the diameter d of the pins, (*b*) the average bearing stress in the link.



Rod AB is in compression.

$A = bt$ where $b = 50\text{mm}$ and $t = 6\text{ mm}$

$A = (0.050)(0.006) = 300 \times 10^{-6}\ \text{m}^2$

$P = -\sigma A = -(-140 \times 10^{6})(300 \times 10^{-6})$
$= 42 \times 10^{3}\ \text{N}$

For the pin, $A_P = \frac{\pi}{4} d^2$ and $\tau = \frac{P}{A_P}$

$A_p = \frac{P}{\tau} = \frac{42 \times 10^3}{80 \times 10^6} = 525 \times 10^{-6}\ \text{m}^2$

(a) Diameter d.

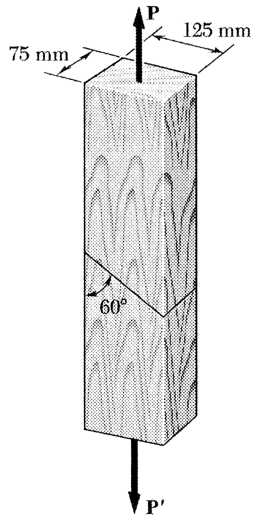
$d = \sqrt{\frac{4A_P}{\pi}} = \sqrt{\frac{(4)(525 \times 10^{-6})}{\pi}} = 2.585 \times 10^{-3}\ \text{m}$

$d = 25.9\ \text{mm}$ ◄

(b) $\sigma_b = \frac{P}{dt} = \frac{42 \times 10^3}{(25.85 \times 10^{-3})(0.006)} = 271 \times 10^{6}\ \text{Pa}$

$\sigma_b = 271\ \text{MPa}$ ◄

Problem 1.29



1.29 The 5.6-kN load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$P = 6227 N$ $\qquad \theta = 90° - 60° = 30°$

$A_o = (0.125)(0.075) = 9.375 \times 10^{-3}\ m^2$

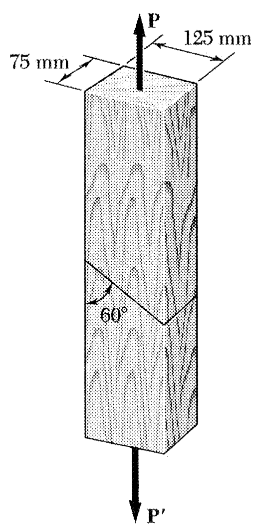
$$\sigma = \frac{P\cos^2\theta}{A_o} = \frac{(6.227 \times 10^3)(\cos 30°)^2}{9.375 \times 10^{-3}}$$

$\sigma = 0.498\ MPa$ ◄

$$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(6.227 \times 10^3)\sin 60°}{(2)(9.375 \times 10^{-3})}$$

$\tau = 0.288\ MPa$ ◄

Problem 1.30



1.30 Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 525 kPa, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding tensile stress in the splive.

$A_o = (0.125)(0.075) = 9.375 \times 10^{-3}\ m^2$

$\theta = 90° - 60° = 30°$

$$\sigma = \frac{P\cos^2\theta}{A_o}$$

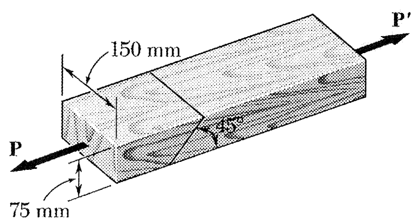
(a) $$P = \frac{\sigma A_o}{\cos^2\theta} = \frac{(525 \times 10^3)(9.375 \times 10^{-3})}{\cos^2 30°} = 6562 N$$

$P = 6.562\ kN$ ◄

(b) $$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(6562)\sin 60°}{(2)(9.375 \times 10^{-3})}$$

$\tau = 0.303\ MPa$ ◄

Problem 1.31



1.31 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

$$\theta = 90° - 45° = 45°$$

$$P = 11 \text{ kN} = 11 \times 10^3 \text{ N}$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

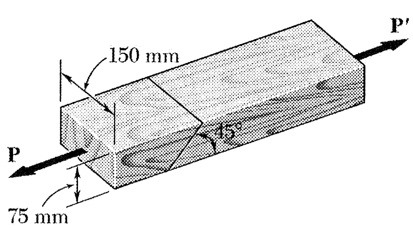
$$\sigma = \frac{P\cos^2\theta}{A_o} = \frac{(11 \times 10^3)\cos^2 45°}{11.25 \times 10^{-3}}$$

$$= 489 \times 10^3 \text{ Pa} \qquad \sigma = 489 \text{ kPA}$$

$$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(11 \times 10^3)(\sin 90°)}{(2)(11.25 \times 10^{-3})}$$

$$= 4.89 \times 10^3 \text{ Pa} \qquad \tau = 489 \text{ kPA}$$

Problem 1.32



1.32 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 560 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding shearing stress in the splice.

$$\theta = 90° - 45° = 45°$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

$$\sigma = 560 \text{ kPa} = 560 \times 10^3 \text{ Pa}$$

$$\sigma = \frac{P\cos^2\theta}{A_o}$$

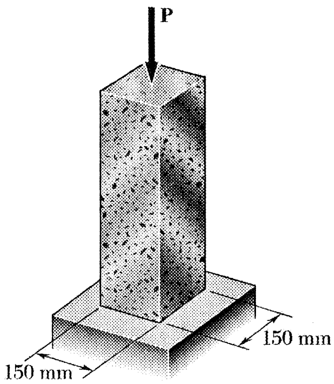
(a)
$$P = \frac{\sigma A_o}{\cos^2\theta} = \frac{(560 \times 10^3)(11.25 \times 10^{-3})}{\cos^2 45°}$$

$$= 12.60 \times 10^3 \text{ N} \qquad P = 12.60 \text{ kN}$$

(b)
$$\tau = \frac{P\sin\theta\cos\theta}{A_o} = \frac{(12.60 \times 10^3)(\sin 45°)(\cos 45°)}{11.25 \times 10^{-3}}$$

$$= 560 \times 10^3 \text{ Pa} \qquad \tau = 560 \text{ kPa}$$

Problem 1.33



1.33 A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 17 MPa, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on the surface, (*d*) the maximum value of the normal stress in the block.

$A_o = (0.15)(0.15) = 0.0225\ m^2 \quad \tau_{max} = 17\ MPa$

$\theta = 45°$ for plane of τ_{max}

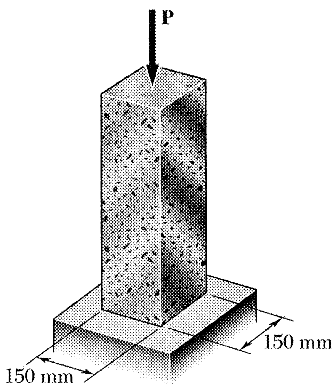
(a) $\tau_{max} = \frac{|P|}{2A_o} \quad \therefore |P| = 2A_o\tau_{max} = (2)(0.0225)(17\times10^6)$
$= 765\ kN$

(b) $\sin 2\theta = 1 \quad 2\theta = 90° \quad \theta = 45°$

(c) $\sigma_{45} = \frac{P}{A_o}\cos^2 45° = \frac{P}{2A_o} = \frac{-765\times10^3}{2(0.0225)} = -17\ MPa$

(d) $\sigma_{max} = \frac{P}{A_o} = -\frac{765\times10^3}{0.0225} = -34\ MPa$

Problem 1.34



1.34 A 960-kN load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

$A_o = (0.15)(0.15) = 0.0225\ m^2$

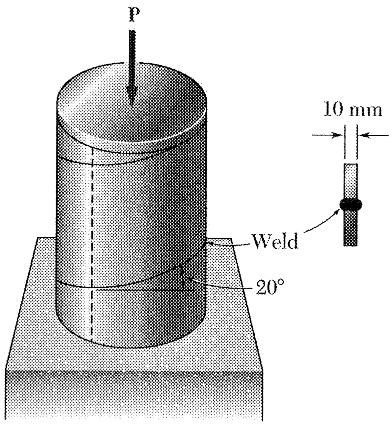
$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{-960\times10^3}{0.0225}\cos^2\theta = -42.67\times10^6\cos^2\theta$

(a) max tensile stress = 0 at $\theta = 90°$

max. compressive stress = 42.7 MPa
at $\theta = 0°$

(b) $\tau_{max} = \frac{P}{2A_o} = \frac{960\times10^3}{2(0.0225)} = 21.3\ MPa$
at $\theta = 45°$

Problem 1.35



1.35 A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 60$ MPa and $\tau = 36$ MPa, determine the magnitude P of the largest axial force that can be applied to the pipe.

$d_o = 0.400$ m $\quad r_o = \frac{1}{2}d_o = 0.200$ m

$r_i = r_o - t = 0.200 - 0.010 = 0.190$ m

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2)$

12.25×10^{-3} m^2

$\theta = 20°$

Based on $|\sigma| = 60$ MPa: $\quad \sigma = \frac{P}{A_o}\cos^2\theta$

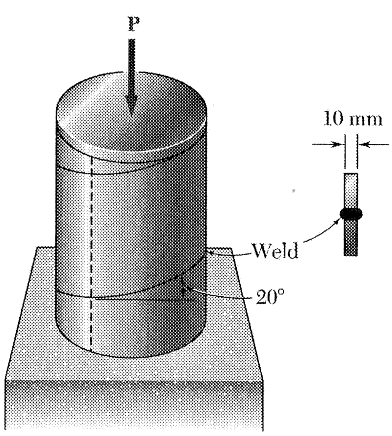
$$P = \frac{A_o\sigma}{\cos^2\theta} = \frac{(12.25\times10^{-3})(60\times10^6)}{\cos^2 20°} = 833\times10^3 \text{ N}$$

Based on $|\tau| = 30$ MPa $\quad \tau = \frac{P}{2A_o}\sin 2\theta$

$$P = \frac{2A_o\tau}{\sin 2\theta} = \frac{(2)(12.25\times10^{-3})(36\times10^6)}{\sin 40°} = 1372\times10^3 \text{ N}$$

Smaller value is the allowable value of P. $\quad P = 833$ kN

Problem 1.36



1.36 A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 300-kN axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

$d_o = 0.400$ m $\quad r_o = \frac{1}{2}d_o = 0.200$ m

$r_i = r_o - t = 0.200 - 0.010 = 0.190$ m

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2)$

$= 12.25 \times 10^{-3}$ m^2

$\theta = 20°$

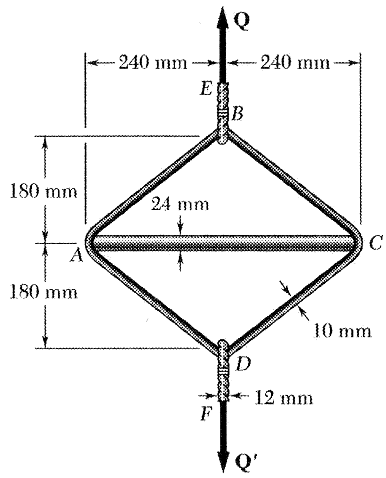
$$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{-300\times10^3\cos^2 20°}{12.25\times10^{-3}}$$

$= -21.6\times10^6$ Pa $\quad \sigma = -21.6$ MPa

$$\tau = \frac{P}{2A_o}\sin 2\theta = \frac{-300\times10^3 \sin 40°}{(2)(12.25\times10^{-3})}$$

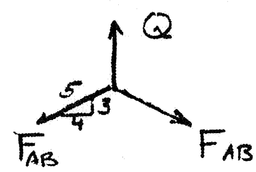
$= -7.87\times10^6$ $\quad \tau = 7.87$ MPa

Problem 1.37



1.37 A steel loop *ABCD* of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of 12-mm diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

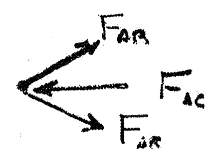
Using joint B as a free body and considering symmetry,



$$2 \cdot \frac{3}{5} F_{AB} - Q = 0$$

$$Q = \frac{6}{5} F_{AB}$$

Using joint A as a free body and considering symmetry,



$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$

Based on strength of cable BE.

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop.

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N}$$

Based on strength of rod AC.

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2$$

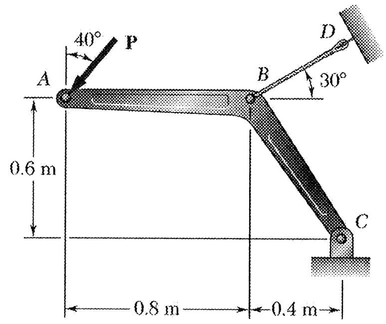
$$= \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N}$$

Actual ultimate load Q_U is the smallest. $\therefore$ $Q_U = 45.24 \times 10^3$ N

Allowable load $Q = \frac{Q_U}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3$ N

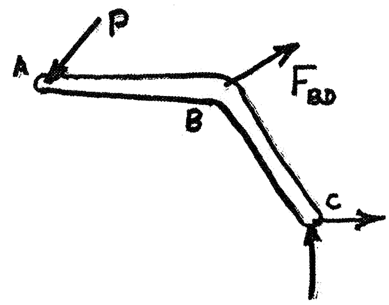
$Q = 15.08$ kN

Problem 1.38



1.38 Member *ABC*, which is supported by a pin and bracket at *C* and a cable *BD*, was designed to support the 16-kN load **P** as shown. Knowing that the ultimate load for cable *BD* is 100 kN, determine the factor of safety with respect to cable failure.

Use member ABC as a free body, and note that member BD is a two-force member.



$\circlearrowleft \Sigma M_c = 0:$

$$(P\cos 40^\circ)(1.2) + (P\sin 40^\circ)(0.6) - (F_{BD}\cos 30^\circ)(0.6) - (F_{BD}\sin 30^\circ)(0.4) = 0$$

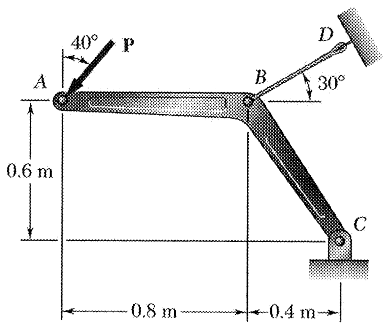
$$1.30493\,P - 0.71962\,F_{BD} = 0$$

$$F_{BD} = 1.81335\,P = (1.81335)(16\times10^3) = 2.9014\times10^3\ \text{N}$$

$$F_{ult} = 100\times10^3\ \text{N}$$

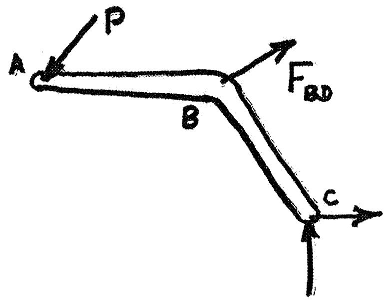
$$F.S. = \frac{F_{ult}}{F_{BD}} = \frac{100\times10^3}{2.9014\times10^3} \qquad F.S. = 3.45$$

Problem 1.39



1.39 Knowing that the ultimate load for cable *BD* is 100 kN and that a factor of safety of 3.2 with respect to cable failure is required, determine the magnitude of the largest force **P** which can be safely applied as shown to member *ABC*.

Use member ABC as a free body, and note that member BD is a two-force member.



$\circlearrowleft \Sigma M_c = 0:$

$$(P\cos 40^\circ)(1.2) + (P\sin 40^\circ)(0.6) - (F_{BD}\cos 30^\circ)(0.6) - (F_{BD}\sin 30^\circ)(0.4) = 0$$

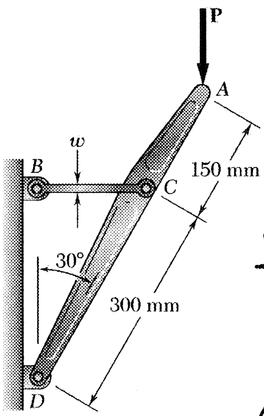
$$1.30493\,P - 0.71962\,F_{BD} = 0$$

$$P = 0.55404\,F_{BD}$$

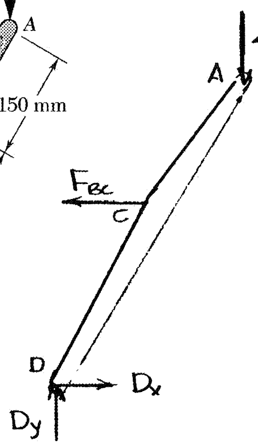
Allowalbe value of F_{BD}. $\quad F_{BD} = \dfrac{F_{ult}}{F.S.} = \dfrac{100\ \text{kN}}{3.2} = 3.125\ \text{kN}$

$$P_{all} = (0.55404)(3.125) \qquad P_{all} = 1.732\ \text{kN}$$

Problem 1.40



1.40 The horizontal link BC is 6 mm thick, has a width w = 30 mm, and is made of a steel with a 450-MPa ultimate strength in tension. What is the factor of safety if the structure shown is designed to support a load P = 40 kN?



$$+\circlearrowright \Sigma M_C = 0$$

$$(0.3 \cos 30°) F_{AB} - (0.45 \sin 30°)(40) = 0$$

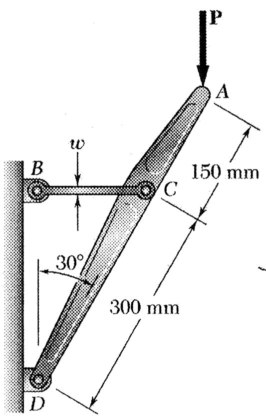
$$F_{BC} = 34.6 \text{ kN}$$

$$A_{BC} = (0.006)(0.03) = 1.8 \times 10^{-4} \text{ m}^2$$

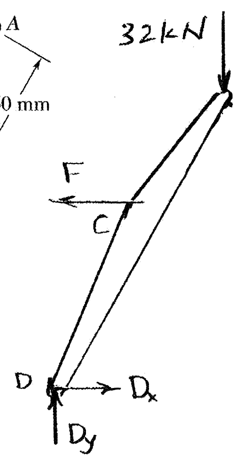
$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{\sigma_{ult}}{F.S}$$

$$F.S = \frac{A_{BC}\,\sigma_{ult}}{F_{BC}} = \frac{(1.8 \times 10^{-4})(450 \times 10^{6})}{34.6 \times 10^{3}} = 2.34$$

Problem 1.41



1.41 The horizontal link BC is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be the width w of the link if the structure shown is to be designed to support a load P = 32 kN with a factor of safety equal to 3?



$$+\circlearrowright \Sigma M_C = 0$$

$$(0.3 \cos 30°) F_{AB} - (0.45 \sin 30°)(32) = 0$$

$$F_{AB} = 27.7 \text{ kN}$$

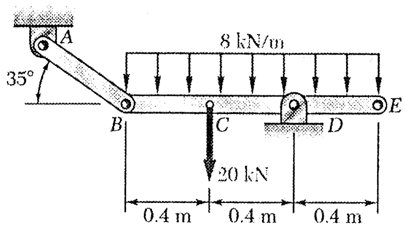
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{F_{AB}}{tw} = \frac{\sigma_{ult}}{F.S.}$$

$$w = \frac{(F.S.)\,F_{AB}}{t\,\sigma_{ult}} = \frac{(3)(27.7 \times 10^{3})}{(0.006)(450 \times 10^{6})}$$

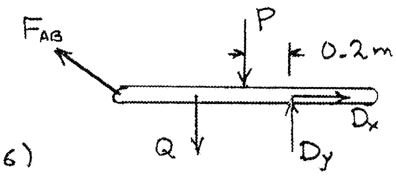
$$w = 0.03078 \text{ m}$$

$$= 30.8 \text{ mm}$$

Problem 1.42



1.42 Link *AB* is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for *AB* for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at *A* and *B*.



$P = (1.2)(8) = 9.6 \text{ kN}$

$+\circlearrowleft \Sigma M_D = 0:$

$-(0.8)(F_{AB} \sin 35°) + (0.2)(9.6) + (0.4)(20) = 0$

$F_{AB} = 21.619 \text{ kN} = 21.619 \times 10^3 \text{ N}$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{ult}}{F.S.}$$

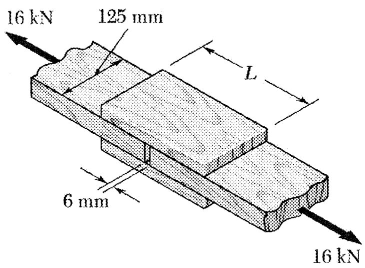
$$A_{AB} = \frac{(F.S.)F_{AB}}{\sigma_{ult}} = \frac{(3.50)(21.619 \times 10^3)}{450 \times 10^6}$$

$= 168.1 \times 10^{-6} \text{ m}^2$

$A_{AB} = 168.1 \text{ mm}^2$

Problem 1.43

1.43 The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.



There are 4 separate areas of glue. Each glue area must transmit 8 kN of shear load.

$$P = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

Required ultimate load.

$$P_U = (F.S.)P = (2.75)(8 \times 10^3) = 22 \times 10^3 \text{ N}$$

Required length of each glue area.

$$P_U = \tau_U A = \tau_U l w \qquad l = \frac{P_U}{\tau_U w} = \frac{22 \times 10^3}{(2.5 \times 10^6)(0.125)} = 70.4 \times 10^{-3} \text{ m}$$

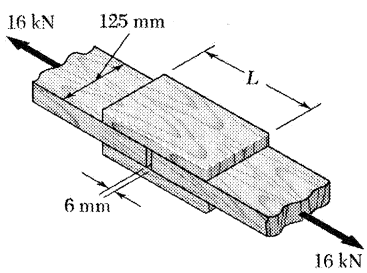
Length of splice: $L = 2l + c = (2)(70.4 \times 10^{-3}) + 0.006 = 0.1468 \times 10^{-3}$ m

$$L = 146.8 \text{ mm}$$

Problem 1.44

1.44 For the joint and loading of Prob. 1.43, determine the factor of safety, knowing that the length of each splice is $L = 180$ mm.

1.43 The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.



There are 4 separate areas of glue. Each glue area must transmit 8 kN of shear load.

$$P = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

Length of splice. $L = 2l + c$ where l = length of glue and c = clearance.

$$l = \tfrac{1}{2}(L - c) = \tfrac{1}{2}(0.180 - 0.006) = 0.087 \text{ m}$$

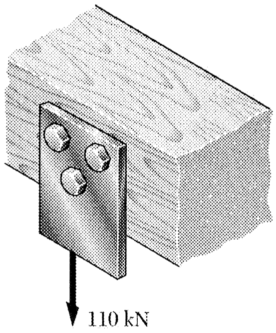
Area of glue. $A = lw = (0.087)(0.125) = 10.875 \times 10^{-3}$ m

Ulimate load. $P_U = \tau_U A = (2.5 \times 10^6)(10.875 \times 10^{-3}) = 27.1875 \times 10^3$ N

Factor of safety. $F.S. = \dfrac{P_U}{P} = \dfrac{27.1875 \times 10^3}{8 \times 10^3}$ $\qquad F.S. = 3.40$

Problem 1.45

1.45 Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.



For each bolt, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2$

$= 254.47 \times 10^{-6} \text{ m}^2$

$P_U = A\tau_U = (254.47 \times 10^{-6})(360 \times 10^6)$

$= 91.609 \times 10^3 \text{ N}$

For the three bolts, $P_U = (3)(91.609 \times 10^3)$

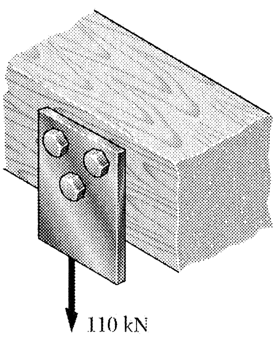
$= 274.83 \times 10^3 \text{ N}$

Factor of safety.

$$F.S. = \frac{P_U}{P} = \frac{274 \times 10^3}{110 \times 10^3} = \qquad F.S. = 2.50$$

Problem 1.46

1.46 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110 kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.



For each bolt, $P = \frac{110}{3} = 36.667 \text{ kN}$

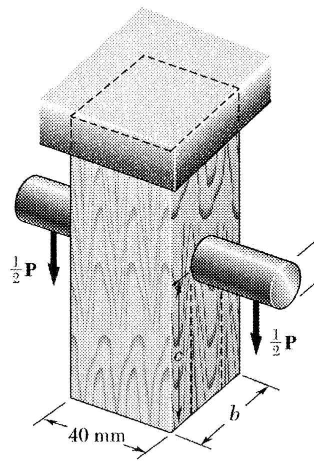
Required $P_U = (F.S.)P = (3.35)(36.667) = 122.83 \text{ kN}$

$$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$$

$$d = \sqrt{\frac{4P_U}{\pi \tau_U}} = \sqrt{\frac{(4)(122.83 \times 10^3)}{\pi(360 \times 10^6)}} = 20.8 \times 10^{-3} \text{ m}$$

$d = 20.8 \text{ mm}$

Problem 1.47



1.47 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

1.48 For the support of Prob. 1.47, knowing that the diameter of the pin is d = 16 mm and that the magnitude of the load is $P = 20$ kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of *b* and *c* if the factor of safety for the wooden members is the same as that found in part *a* for the pin.

$P = 20 \text{ kN} = 20 \times 10^3 \text{ N}$

(a) Pin: $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

Double shear $\quad \tau = \frac{P}{2A} \quad \tau_U = \frac{P_U}{2A}$

$$P_U = 2A\tau_U = (2)(201.16 \times 10^{-6})(145 \times 10^6)$$
$$= 58.336 \times 10^3 \text{ N}$$

$$F.S. = \frac{P_U}{P} = \frac{58.336 \times 10^3}{20 \times 10^3} = 2.92$$

(b) Tension in wood $\quad P_U = 58.336 \times 10^3$ N for same F.S.

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \quad \text{where } w = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{58.336 \times 10^3}{(0.040)(60 \times 10^6)} = 40.3 \times 10^3 \text{ m}$$

$$b = 40.3 \text{ mm}$$

Shear in wood $\quad P_U = 58.336 \times 10^3$ N for same F.S.

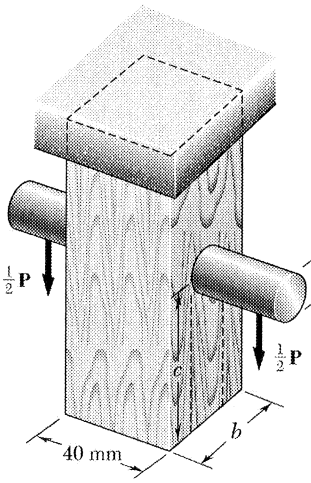
Double shear; each area is $A = wc$

$$\tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$$

$$c = \frac{P_U}{2w\tau_U} = \frac{58.336 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 97.2 \times 10^{-3} \text{ m}$$

$$c = 97.2 \text{ mm}$$

Problem 1.48



1.48 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

Based on double shear in pin

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$

$$= \frac{\pi}{4}(2)(0.012)^2(145\times10^6) = 32.80\times10^3 \text{ N}$$

Based on tension in wood

$$P_U = A\sigma_U = w(b-d)\sigma_U$$

$$= (0.040)(0.040-0.012)(60\times10^6)$$

$$= 67.2\times10^3 \text{ N}$$

Based on double shear in the wood

$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$

$$= 33.0\times10^3 \text{ N}$$

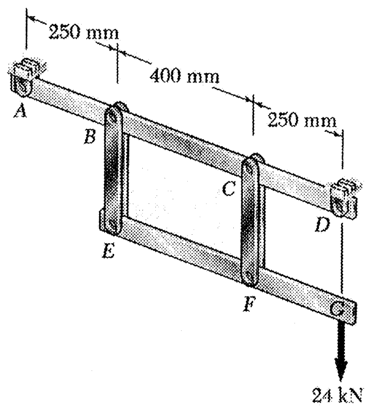
Use smallest $\qquad P_U = 32.8\times10^3$ N

Allowable $\qquad P = \frac{P_U}{F.S.} = \frac{32.8\times10^3}{3.2} = 10.25\times10^3$ N

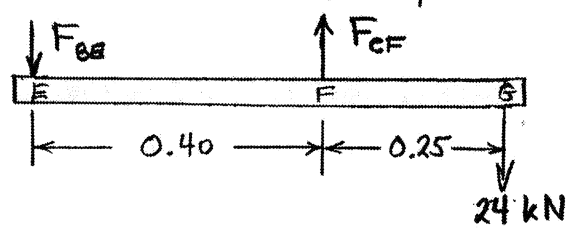
10.25 kN

Problem 1.49

1.49 Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a 10 × 40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at *C* and *F* has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.



Use member EFG as free body.



$\curvearrowright \Sigma M_E = 0$

$0.40\, F_{CF} - (0.65)(24 \times 10^3) = 0$

$F_{CF} = 39 \times 10^3$ N

Based on tension in links CF

$A = (b-d)t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6}\ \text{m}^2$ (one link)

$F_U = 2\sigma_U A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3$ N

Based on double shear in pins

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ \text{m}^2$

$F_U = 2\tau_U A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3$ N

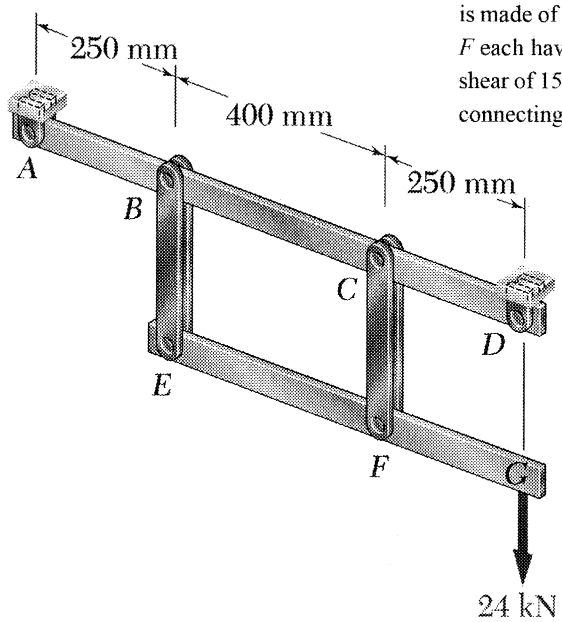
Actual F_U is smaller value, i.e. $F_U = 94.248 \times 10^3$ N

Factor of safety $\quad F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{94.248 \times 10^3}{39 \times 10^3} = 2.42$ ◄

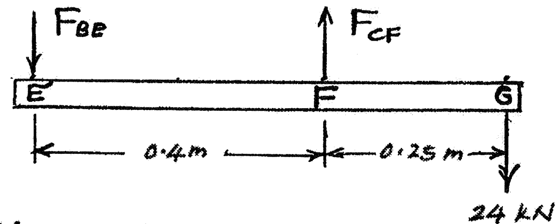
Problem 1.50

1.50 Solve Prob. 1.49, assuming that the pins at *C* and *F* have been replaced by pins with a 30 mm diameter.

1.49 Each of the two vertical links CF connecting the two horizontal members *AD* and *EG* has a uniform rectangular cross section 10 mm thick and 40 mm wide, and is made of a steel with an ultimate strength in tension of 400 MPa. The pins at *C* and *F* each have a 20 mm diameter and are made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.



Use member EFG as free body.



$\circlearrowleft \Sigma M_E = 0:$

$$0.4\, F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \text{ kN}$$

Failure by tension in links CF. (2 parallel links)

Net section area for 1 link: $A = (b-d)t = (0.04 - 0.02)(0.01) = 200 \times 10^{-6}\, m^2$

$$F_U = 2A\sigma_U = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160 \text{ kN}$$

Failure by double shear in pins.

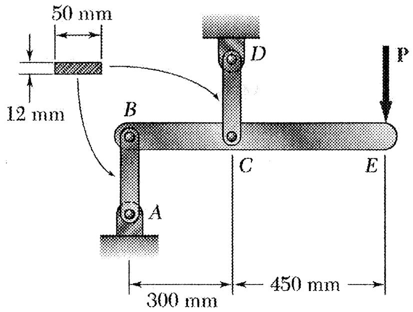
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.02)^2 = 314.16 \times 10^{-6}\, m^2$$

$$F_U = 2A\tau_U = (2)(314.16 \times 10^{-6})(150 \times 10^6) = 94.25 \text{ kN}$$

Actual ultimate load is the smaller value. $F_U = 94.25$ kN

<u>Factor of safety:</u> $F.S. = \frac{F_U}{F_{CF}} = \frac{94.25}{39}$ $\quad F.S. = 2.42$

Problem 1.51

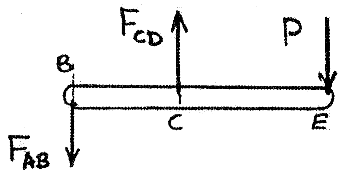


1.51 Each of the steel links *AB* and *CD* is connected to a support and to member *BCE* by 25-mm-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 210 MPa for the steel used in the pins and that the ultimate normal stress is 490 MPa for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.0 is desired. (Note that the links are not reinforced around the pin holes.)

Use member BCE as free body.

$\circlearrowleft + \Sigma M_B = 0: \quad 0.3 F_{CD} - 0.75 P = 0$

$$P = \frac{2}{5} F_{CD}$$



$\circlearrowleft + \Sigma M_C = 0: \quad 0.3 F_{AB} - 0.45 P = 0$

$$P = \frac{2}{3} F_{AB}$$

Both links have the same area, pin diameter and material, Therefore, they have the same ultimate load.

Failure by pin in single shear. $\quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.025)^2 = 490.9 \times 10^{-6}\ m^2$

$$F_U = \tau_U A = (210 \times 10^6)(490.9 \times 10^{-6}) = 103.09\ kN$$

Failure by tension in link. $\quad A = (b-d)t = (0.05 - 0.025)0.012 = 3 \times 10^{-4}\ m^2$

$$F_U = \sigma_U A = (490 \times 10^6)(3 \times 10^{-4}) = 147\ kN$$

Ultimate load for link and pin is the smaller.

$$F_U = 103.09\ kN$$

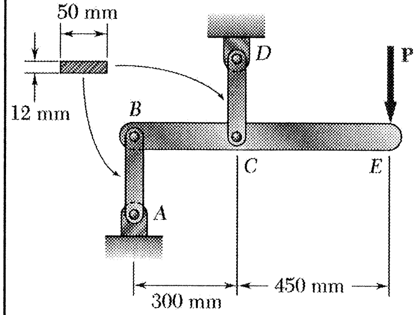
Allowable values of F_{CD} and F_{AB}.

$$F_{all} = \frac{F_U}{F.S.} = \frac{103.09}{3.0} = 34.36\ kN$$

Allowable load for structure is the smaller of $\frac{2}{3} F_{all}$ and $\frac{2}{5} F_{all}$.

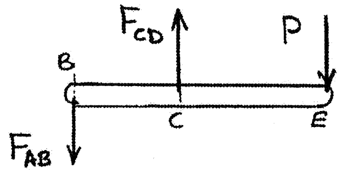
$$P = \frac{2}{5}(34.36) \qquad P = 13.7\ kN$$

Problem 1.52



1.52 An alternative design is being considered to support member *BCE* of Prob. 1.51, in which link *CD* will be replaced by two links, each of 6×50-mm cross section, causing the pins at *C* and *D* to be in double shear. Assuming that all other specifications remain unchanged, determine the allowable load P if an overall factor of safety of 3.0 is desired.

1.51 Each of the steel links *AB* and *CD* is connected to a support and to member *BCE* by 25-mm-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 210 MPa for the steel used in the pins and that the ultimate normal stress is 490 MPa for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.0 is desired. (Note that the links are not reinforced around the pin holes.)



Use member BCE as free body.

$+\circlearrowleft \Sigma M_B = 0$: $0.3 F_{CD} - 0.75 P = 0$

$P = \frac{2}{5} F_{CD}$

$+\circlearrowleft \Sigma M_C = 0$: $0.3 F_{AB} - 0.45 P = 0$

$P = \frac{2}{3} F_{AB}$

Area of all pins: $A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.025)^2 = 490.9\times10^{-6}\ m^2$

Net section area of link AB: $A_{net} = (b-d)t_{AB} = (0.05-0.025)(0.012) = 3\times10^{-4}\ m^2$

Net section area of the 2 links CD is the same.

Failure by pins A and B in single shear. $(F_{AB})_U = \tau_U A_{pin}$

$(F_{AB})_U = (210\times10^6)(490.9\times10^{-6}) = 103.09\ kN$

Failure by tension in link AB. $(F_{AB})_U = \sigma_U A_{net}$

$(F_{AB})_U = (490\times10^6)(3\times10^{-4}) = 147\ kN$

Ultimate load for link and pins AB is the smaller: $(F_{AB})_U = 103.09\ kN$

Corresponding ultimate load, $P_U = \frac{2}{3}(F_{AB})_U = 68.73\ kN$

Failure by pins C and in double shear. $(F_{CD})_U = 2\tau_U A_{pin}$

$(F_{CD})_U = (2)(210\times10^6)(490.9\times10^{-6}) = 206.18\ kN$

Failure by tension in links CD. $(F_{CD})_U = \sigma_U A_{net}$

$(F_{CD})_U = (490\times10^6)(3\times10^{-4}) = 147\ kN$

Ultimate load for links and pins CD is the smaller; $(F_{CD})_U = 147\ kN$

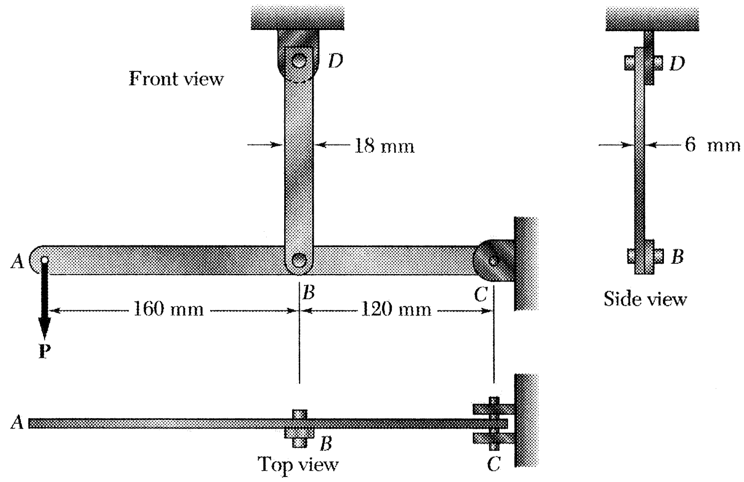
Corresponding ultimate load: $P_U = \frac{2}{5}(F_{CD})_U = 58.8\ kN$

Actual ultimate load is the smaller. $P_U = 58.8\ kN$

<u>Allowable load P:</u> $P = \frac{P_U}{F.S.} = \frac{58.8}{3.0}$ $P = 19.6\ kN$

Problem 1.53

1.53 In the steel structure shown, a 6-mm-diameter pin is used at *C* and 10-mm-diameter pins are used at *B* and *D*. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link *BD*. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at *A*. Note that link *BD* is not reinforced around the pin holes.



Use free body ABC.

$+\circlearrowleft \Sigma M_C = 0:$

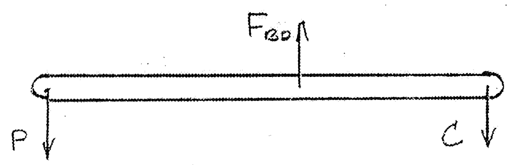
$0.280\,P - 0.120\,F_{BD} = 0$

$P = \frac{3}{7} F_{BD} \qquad (1)$

$+\circlearrowleft \Sigma M_B = 0:$

$0.160\,P - 0.120\,C = 0$

$P = \frac{3}{4} C \qquad (2)$



Tension on net section of link BD.

$$F_{BD} = \sigma A_{net} = \frac{\sigma_U}{F.S.} A_{net}$$

$$= \left(\frac{400 \times 10^6}{3}\right)(6 \times 10^{-3})(18 - 10)(10^{-3})$$

$$= 6.40 \times 10^3 \text{ N}$$

Shear in pins at B and D.

$$F_{BD} = \tau A_{pin} = \frac{\tau_U}{F.S.} \frac{\pi}{4} d^2 = \left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(10 \times 10^{-3})^2 = 3.9270 \times 10^3 \text{ N}$$

Smaller value of F_{BD} is 3.9270×10^3 N.

From (1) $\quad P = \left(\frac{3}{7}\right)(3.9270 \times 10^3) = 1.683 \times 10^3$ N

Shear in pin at C

$$C = 2\tau A_{pin} = 2 \frac{\tau_U}{F.S.} \frac{\pi}{4} d^2 = (2)\left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(6 \times 10^{-3})^2 = 2.8274 \times 10^3 \text{ N}$$

From (2) $\quad P = \left(\frac{3}{4}\right)(2.8274 \times 10^3) = 2.12 \times 10^3$ N

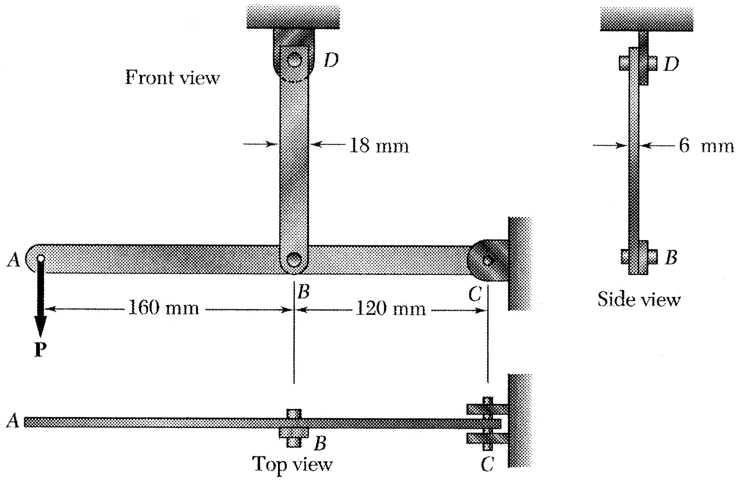
Smaller value of P is allowable value. $\qquad P = 1.683 \times 10^3$ N

$P = 1.683$ kN

Problem 1.54

1.54 Solve Prob. 1.53, assuming that the structure has been redesigned to use 12-mm-diameter pins at B and D and no other change has been made.

1.53 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at A. Note that link BD is not reinforced around the pin holes.



Use free body ABC.

$+\circlearrowleft \Sigma M_C = 0:$

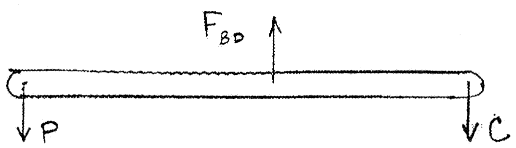
$0.280 P - 0.120 F_{BD} = 0$

$P = \frac{3}{7} F_{BD}$ (1)

$+\circlearrowleft \Sigma M_B = 0:$

$0.160 P - 0.120 C = 0$

$P = \frac{3}{4} C$ (2)



Tension on net section of link BD.

$$F_{BD} = \sigma A_{net} = \frac{\sigma_U}{F.S.} A_{net}$$

$$= \left(\frac{400 \times 10^6}{3}\right)(6 \times 10^{-3})(18 - 12)(10^{-3})$$

$$= 4.80 \times 10^3 \text{ N}$$

Shear in pins at B and D.

$$F_{BD} = \tau A_{pin} = \frac{\tau_U}{F.S.} \frac{\pi}{4} d^2 = \left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(12 \times 10^{-3})^2 = 5.6549 \times 10^3 \text{ N}$$

Smaller value of F_{BD} is 4.80×10^3 N.

From (1), $P = \left(\frac{3}{7}\right)(4.80 \times 10^3) = 2.06 \times 10^3$ N

Shear in pin at C.

$$C = 2\tau A_{pin} = 2 \frac{\tau_U}{F.S.} \frac{\pi}{4} d^2 = (2)\left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(6 \times 10^{-3})^2 = 2.8274 \times 10^3 \text{ N}$$

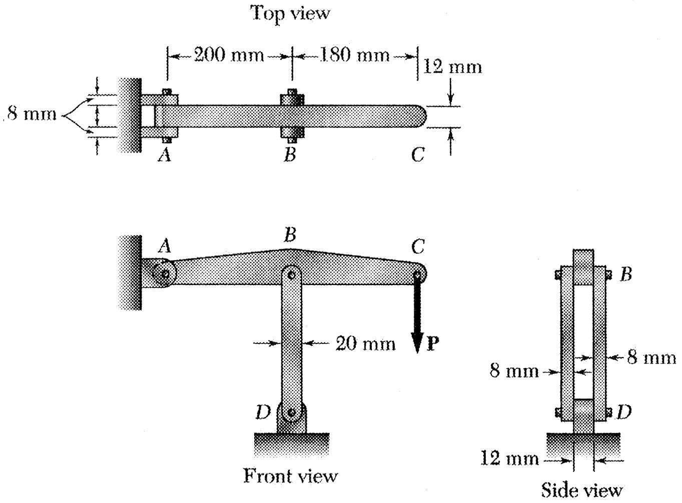
From (2), $P = \left(\frac{3}{4}\right)(2.8274 \times 10^3) = 2.12 \times 10^3$ N

Smaller value of P is the allowable value. $P = 2.06 \times 10^3$ N

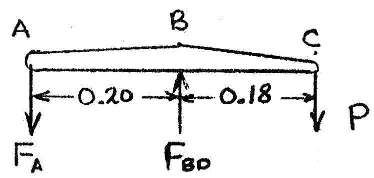
$P = 2.06$ kN

Problem 1.55

1.55 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



Statics: Use ABC as free body.



$\circlearrowleft \Sigma M_B = 0: \quad 0.20\,F_A - 0.18\,P = 0$

$P = \frac{10}{9} F_A$

$\circlearrowleft \Sigma M_A = 0: \quad 0.20\,F_{BD} - 0.38\,P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\ m^2$$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(50.266 \times 10^{-6})}{3.0} = 3.351 \times 10^3\ N$$

$$P = \frac{10}{9} F_A = 3.72 \times 10^3\ N$$

Based on double shear in pins at B and D.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ m^2$$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\ N$$

$$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\ N$$

Based on compression in links BD.

For one link $\quad A = (0.020)(0.008) = 160 \times 10^{-6}\ m^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\ N$$

$$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\ N$$

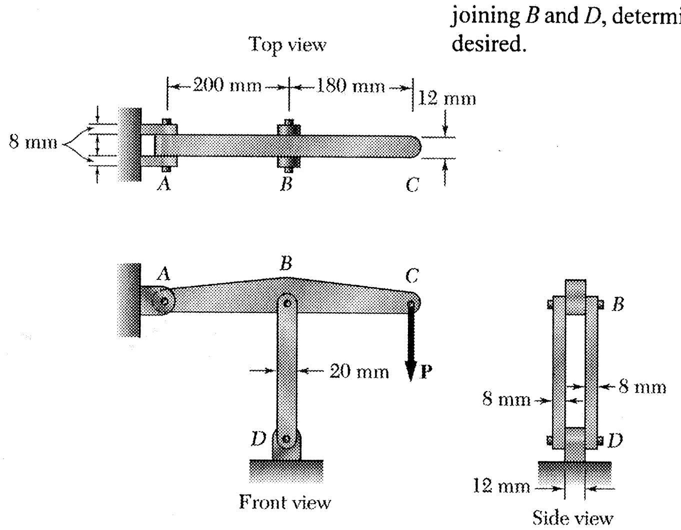
Allowable value of P is smallest. $\therefore \quad P = 3.72 \times 10^3\ N$

$P = 3.72\ kN$

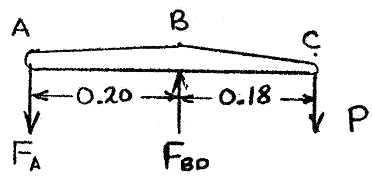
Problem 1.56

1.56 In an alternative design for the structure of Prob. 1.55, a pin of 10-mm-diameter is to be used at A. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

1.55 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



Statics: Use ABC as free body.



$\circlearrowleft \Sigma M_B = 0: \quad 0.20\, F_A - 0.18\, P = 0$

$P = \frac{10}{9} F_A$

$\circlearrowleft \Sigma M_A = 0: \quad 0.20\, F_{BD} - 0.38\, P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ m^2$

$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(78.54 \times 10^{-6})}{3.0} = 5.236 \times 10^3\ N$

$P = \frac{10}{9} F_A = 5.82 \times 10^3\ N$

Based on double shear in pins at B and D.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ m^2$

$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\ N$

Based on compression in links BD.

For one link $\quad A = (0.020)(0.008) = 160 \times 10^{-6}\ m^2$

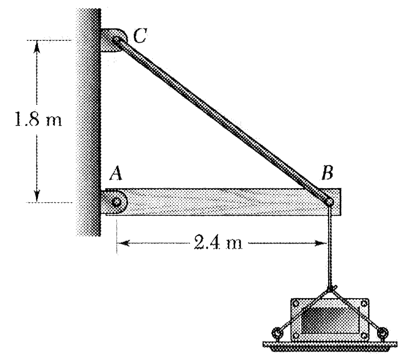
$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\ N$

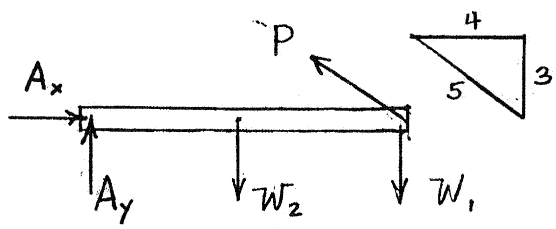
Allowable value of P is smallest. $\therefore \quad P = 3.97 \times 10^3\ N$

$P = 3.97\ kN$

Problem 1.57



*1.57 A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (*a*) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (*b*) What is the corresponding conventional factor of safety for rod BC?



$+\circlearrowleft \Sigma M_A = 0: \quad (2.4)\frac{3}{5}P - 2.4\,W_1 - 1.2\,W_2 \quad \therefore \quad P = \frac{5}{3}W_1 + \frac{5}{6}W_2$

For dead loading, $W_1 = (40)(9.81) = 392.4$ N

$W_2 = (50)(9.81) = 490.5$ N

$P_D = (\frac{5}{3})(392.4) + (\frac{5}{6})(490.5) = 1.0628 \times 10^3$ N

For live loading, $W_1 = mg \qquad W_2 = 0$

$P_L = \frac{5}{3}mg \quad$ from which $\quad m = \frac{3}{5}\frac{P_L}{g}$

Design criterion.

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12\times10^3) - (1.25)(1.0628\times10^{-3})}{1.6}$$

$$= 5.920\times10^3 \text{ N}$$

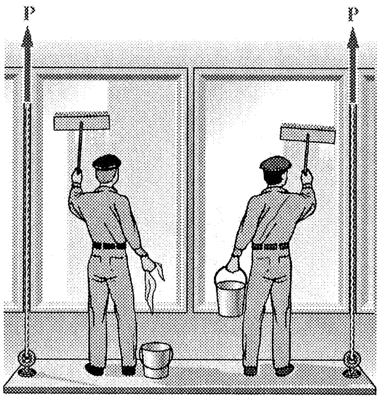
(a) Allowable load. $\quad m = \frac{3}{5}\,\frac{5.92\times10^3}{9.81} \qquad m = 362$ kg ◄

Conventional factor safety.

$$P = P_D + P_L = 1.0628\times10^3 + 5.920\times10^3 = 6.983\times10^3 \text{ N}$$

(b) $\text{F.S.} = \frac{P_U}{P} = \frac{12\times10^3}{6.983\times10^3} \qquad \text{F.S.} = 1.718$ ◄

Problem 1.58



***1.58** The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 72 kg and each of the window washers is assumed to weigh 88 kg with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\varphi}$$

$$= \frac{(1.2)(\frac{1}{2}\times 72) + (1.5)(\frac{3}{4}\times 2\times 88)}{0.85}$$

$$= 283.76 \text{ kg} = 2.78 \text{ kN}$$

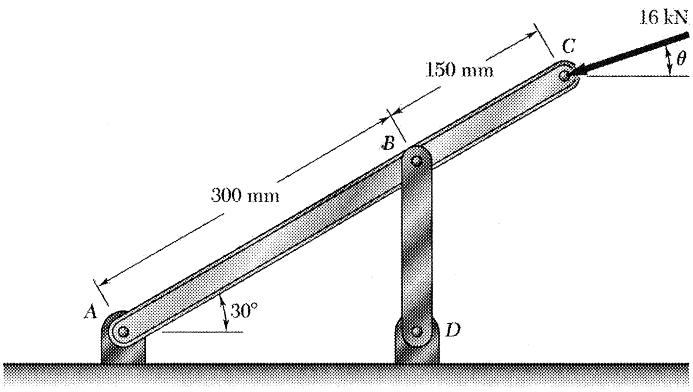
Conventional factor of safety

$$P = P_D + P_L = \tfrac{1}{2}\times 72 + 0.75\times 2\times 88 = 168 \text{ kg} = 1.648 \text{ kN.}$$

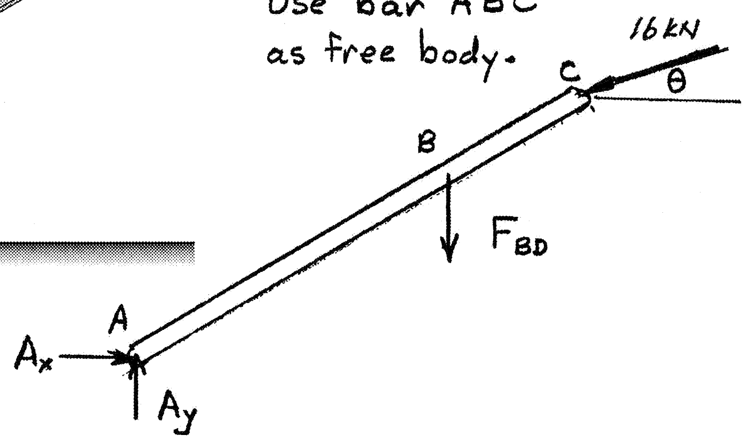
$$\text{F.S.} = \frac{P_U}{P} = \frac{2.784}{1.648} = 1.69$$

Problem 1.59

1.59 Link *BD* consists of a single bar 24 mm wide and 12 mm thick. Knowing that each pin has a 9 mm diameter, determine the maximum value of the average normal stress in link *BD* if (*a*) $\theta = 0$, (*b*) $\theta = 90°$.



Use bar ABC as free body.



(a) $\underline{\theta = 0}$.

$$+\circlearrowleft \Sigma M_A = 0: \quad (0.45 \sin 30°)(16) - (0.3 \cos 30°) F_{BD} = 0$$

$F_{BD} = 13.856$ kN (tension)

Area for tension loading: $A = (b - d)t = (24 - 9)(12) = 180 \text{ mm}^2$

Stress: $\sigma = \dfrac{F_{BD}}{A} = \dfrac{13.856 \times 10^3}{180 \times 10^{-6}}$ $\qquad \sigma = 77 \text{ MPa}$ ◄

(b) $\underline{\theta = 90°}$.

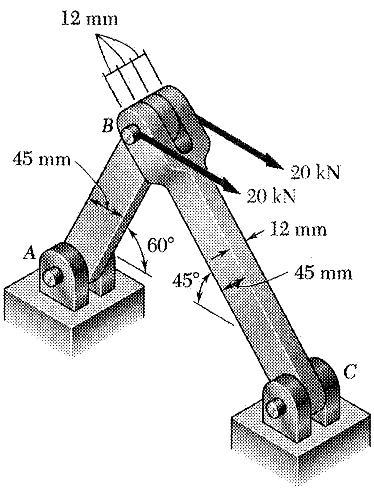
$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.45 \cos 30°)(16) - (0.3 \cos 30°) F_{BD} = 0$$

$F_{BD} = -24$ kN i.e. compression.

Area for compression loading: $A = bt = (24)(12) = 288 \text{ mm}^2$

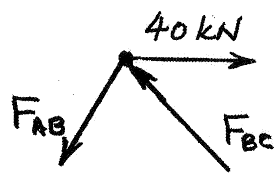
Stress: $\sigma = \dfrac{F_{BD}}{A} = \dfrac{-24000}{288 \times 10^{-6}}$ $\qquad \sigma = -83.3$ kN ◄

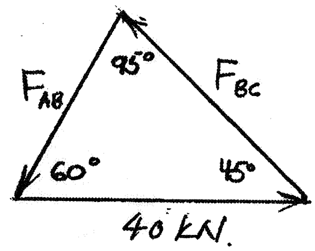
Problem 1.60



1.60 Two horizontal 20 kN forces are applied to pin *B* of the assembly shown. Knowing that a pin of 20 mm diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

Use joint B as free body.





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{40}{\sin 95^\circ}$$

$F_{AB} = 28.4 \text{ kN}$ $\qquad$ $F_{BC} = 34.8 \text{ kN}$

Link AB is a tension member

Minimum section at pin $\quad A_{net} = (0.045 - 0.02)(0.012) = 300 \times 10^{-6} \text{ m}^2$

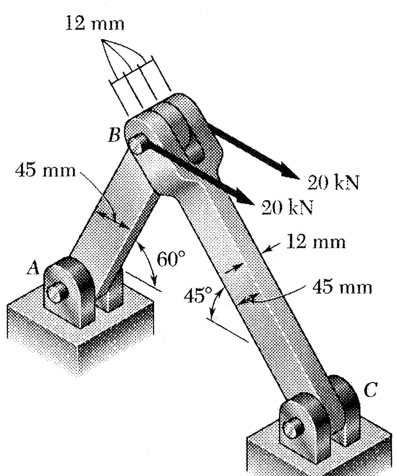
(a) Stress in AB $\quad \sigma_{AB} = \dfrac{F_{AB}}{A_{net}} = \dfrac{28.4}{300 \times 10^{-6}} = 94.7 \text{ MPa}$ ◄

Link BC is a compression member

Cross sectional area is $\quad A = (0.045)(0.012) = 540 \times 10^{-6} \text{ m}^2$

(b) Stress in BC $\quad \sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{34.8}{540 \times 10^{-6}} = -64.4 \text{ MPa}$ ◄

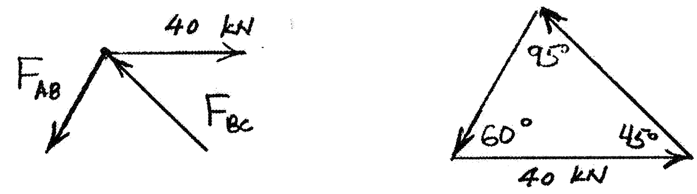
Problem 1.61



1.61 For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.

1.60 Two horizontal 20-kN forces are applied to pin *B* of the assembly shown. Knowing that a pin of 20-mm diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

Use joint B as free body.



Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{40}{\sin 95^\circ} \qquad F_{BC} = 34.77 \text{ kN}$$

(a) Shearing stress in pin at C. $\tau = \frac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4} d^2 = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2$$

$$\tau = \frac{34770}{(2)(314.16 \times 10^{-6})} = 55.338 \times 10^6 \qquad \tau = 55.3 \text{ MPa}$$

(b) Bearing stress at C in member BC. $\sigma_b = \frac{F_{BC}}{A}$

$$A = td = (12)(20) = 240 \text{ mm}^2$$

$$\sigma_b = \frac{34770}{240 \times 10^{-6}} = 144.875 \times 10^6 \qquad \sigma_b = 144.9 \text{ MPa}$$

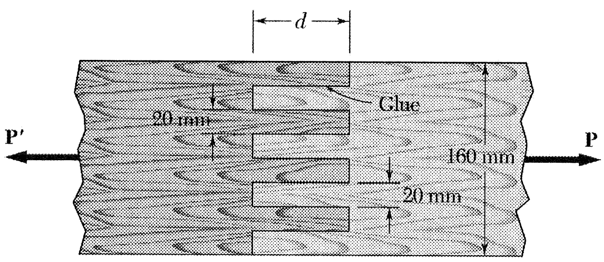
(c) Bearing stress at B in member BC. $\sigma_b = \frac{F_{BC}}{A}$

$$A = 2td = 2(12)(20) = 480 \text{ mm}^2$$

$$\sigma_b = \frac{34770}{480 \times 10^{-6}} = 72.437 \times 10^6 \qquad \sigma_b = 72.4 \text{ MPa}$$

Problem 1.62

1.62 Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.



Seven surfaces carry the total load $P = 7.6$ kN $= 7.6 \times 10^3$.

Let $t = 22$ mm.

Each glue area is $A = dt$

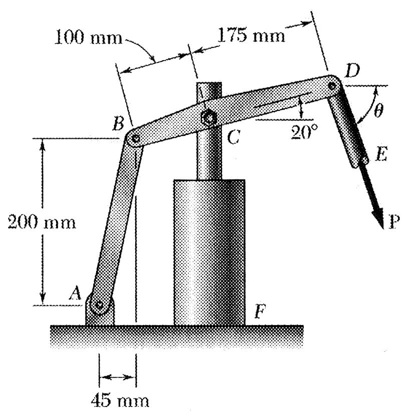
$$\tau = \frac{P}{7A} \qquad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3} \text{ m}^2$$

$$= 1.32404 \times 10^3 \text{ mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2$$

$d = 60.2$ mm ◄

Problem 1.63

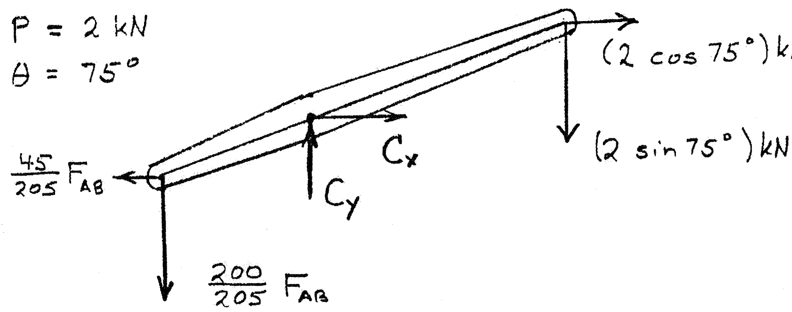


1.63 The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is 15 mm thick and is connected to the vertical rod by a 9-mm-diameter bolt. Knowing that $P = 2$ kN and $\theta = 75°$, determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.

Use member BCD as a free body, and note that AB is a two force member.

$P = 2$ kN

$\theta = 75°$



Length of member AB.

$l_{AB} = \sqrt{200^2 + 45^2}$

$= 205$ mm

$\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{200}{205} F_{AB}\right)(100 \cos 20°) - \left(\frac{45}{205} F_{AB}\right)(100 \sin 20°)$

$- (2 \cos 75°)(175 \sin 20°) - (2 \sin 75°)(175 \cos 20°) = 0$

$84.1696 F_{AB} - 348.668 = 0$ $\qquad F_{AB} = 4.1424$ kN

$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad C_x - \frac{45}{205}(4.1424) + 2 \cos 75° = 0$ $\qquad C_x = 0.3917$ kN

$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{200}{205}(4.1424) - 2 \sin 75° = 0$ $\qquad C_y = 5.9732$ kN

Reaction at C: $\quad C = \sqrt{C_x^2 + C_y^2}$ $\qquad C = 5.9860$ kN

(a) <u>Shearing stress in bolt (single shear).</u>

$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.009)^2 = 63.617 \times 10^{-6}$ m^2

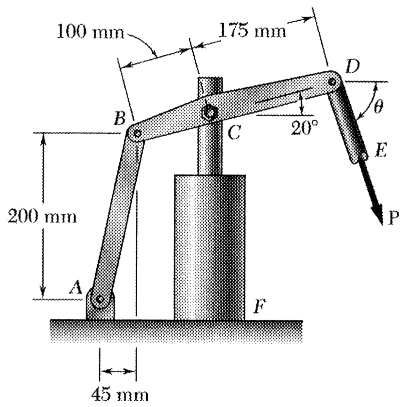
$\tau = \frac{C}{A_{bolt}} = \frac{5.9860 \times 10^3}{63.617 \times 10^{-6}} = 94.09 \times 10^6$ Pa $\qquad \tau = 94.1$ MPa ◄

(b) <u>Bearing stress at C in member BD.</u>

$A_b = dt = (0.009)(0.015) = 135 \times 10^{-6}$ m^2

$\sigma_b = \frac{C}{A_b} = \frac{5.9860 \times 10^3}{135 \times 10^{-6}} = 44.34 \times 10^6$ Pa $\qquad \sigma_b = 44.3$ MPa ◄

Problem 1.64

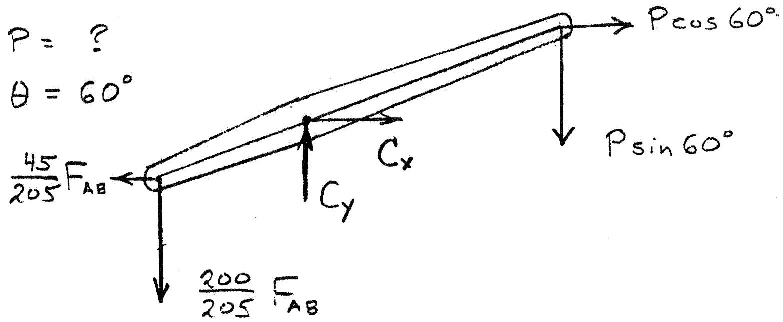


1.64 The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Link *AB* has a uniform rectangular cross section of 12 × 25 mm and is connected at *B* to member *BD* by an 8-mm-diameter pin. Knowing that the maximum allowable average shearing stress in the pin is 140 MPa, determine (*a*) the largest force **P** which may be applied at *E* when $\theta = 60°$, (*b*) the corresponding bearing stress at *B* in link *AB*, (*c*) the corresponding maximum value of the normal stress in link *AB*.

Use member BCD as a free body, and note that AB is a two force member.

$P = ?$

$\theta = 60°$



Length of member AB.

$$l_{AB} = \sqrt{200^2 + 45^2} = 205 \text{ mm}$$

$$+\circlearrowleft \Sigma M_C = 0: \left(\tfrac{200}{205}F_{AB}\right)(100\cos 20°) - \left(\tfrac{45}{205}F_{AB}\right)(100\sin 20°) - (P\cos 60°)(175\sin 20°) - (P\sin 60°)(175\cos 20°) = 0$$

$$84.1696\,F_{AB} - 172.3414\,P = 0 \qquad F_{AB} = 2.0475\,P$$

(a) Allowable load P. Pin at A is in single shear.

$$A_{pin} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.008)^2 = 50.2655\times10^{-6}\ \text{m}^2$$

$$\tau = 140\times10^6\ \text{Pa}$$

$$\tau = \frac{F_{AB}}{A_{pin}} \qquad 140\times10^6 = \frac{2.0475\,P}{50.2655\times10^{-6}}$$

$$P = 3.4370\times10^3\ \text{N} \qquad P = 3.44\ \text{kN}$$

(b) Bearing stress at B in link AB. $d = 8$ mm, $t = 12$ mm

$$A_b = dt = (0.008)(0.012) = 96\times10^{-6}\ \text{m}^2$$

$$F_{AB} = (2.0475)(3.4375\times10^3) = 7.0383\times10^3\ \text{N}$$

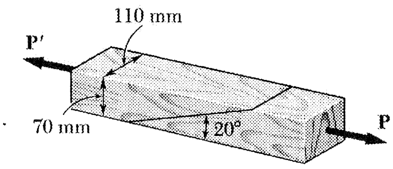
$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{7.0383\times10^3}{96\times10^{-6}} = 73.3\times10^6\ \text{Pa} \qquad \sigma_b = 73.3\ \text{MPa}$$

(c) Maximum normal stress in link AB. $b = 25$ mm, $t = 0.012$ mm

$$A_{net} = (b-d)(t) = (0.025-0.008)(0.012) = 204\times10^{-6}\ \text{m}^2$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{net}} = \frac{7.0383\times10^3}{204\times10^{-6}} = 34.5\times10^6\ \text{Pa} \qquad \sigma_{AB} = 34.5\ \text{MPa}$$

Problem 1.65



1.65 Two wooden members of 70 × 110-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 500 kPa, determine the largest axial load **P** that can be safely applied.

$$A_0 = (0.070\,m)(0.110\,m) = 7.7 \times 10^{-3}\ m^2$$

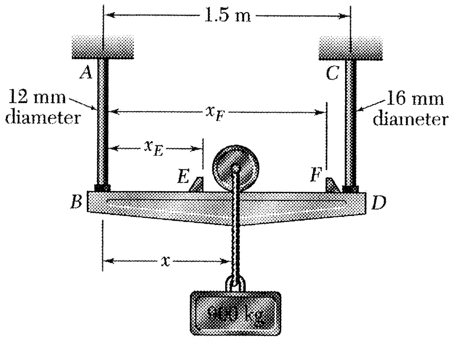
$$\theta = 90° - 20° = 70°$$

$$\tau = \frac{P}{A_0} \sin\theta \cos\theta$$

$$P = \frac{A_0 \tau}{\sin\theta \cos\theta} = \frac{(7.7\times10^{-3})(500\times10^{3})}{\sin 70° \cos 70°} = 11.98\times10^{3}\ N$$

$$P = 11.98\ kN$$

Problem 1.66



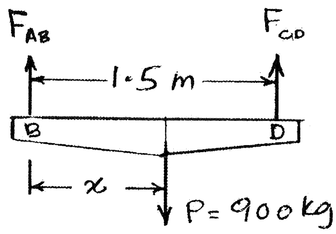
1.66 The 900-kg load may be moved along the beam BD to any position between stops at E and F. Knowing that $\sigma_{all} = 42$ MPa for the steel used in rods AB and CD, determine where the stops should be placed if the permitted motion of the load is to be as large as possible.

Permitted member forces:

$$AB: \quad (F_{AB})_{max} = \sigma_{all} A_{AB} = (42\times10^6)(\tfrac{\pi}{4})(0.012)^2 = 4.75 \text{ kN}$$

$$CD: \quad (F_{CD})_{max} = \sigma_{all} A_{CD} = (42\times10^6)(\tfrac{\pi}{4})(0.016)^2 = 8.44 \text{ kN}$$

Use member BEFD as a free body.



$$P = 900 \text{ kg} = 8.829 \text{ kN}$$

$$+\circlearrowleft \Sigma M_D = 0$$

$$-(1.5)F_{AB} + (1.5 - x_E)P = 0$$

$$1.5 - x_E = \frac{1.5 F_{AB}}{P} = \frac{(1.5)(4.75\times10^3)}{8.829\times10^3} = 0.807$$

$$x_E = 0.693 \text{ m} \blacktriangleleft$$

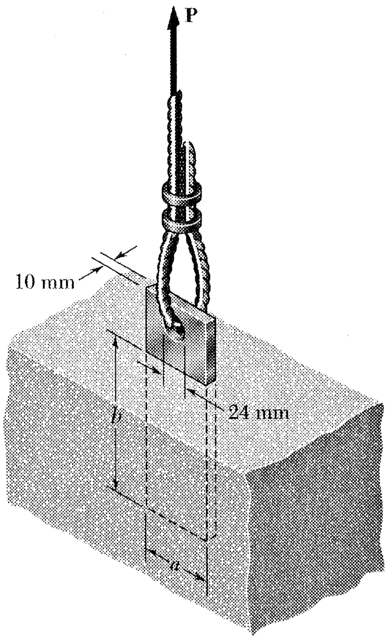
$$+\circlearrowleft \Sigma M_B = 0$$

$$1.5 F_{CD} - xP = 0$$

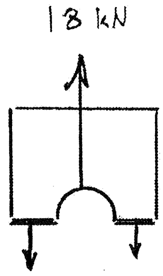
$$x_F = \frac{1.5 F_{CD}}{P} = \frac{(1.5)(8.44)}{8.829}$$

$$x_F = 1.43 \text{ m} \blacktriangleleft$$

Problem 1.67



1.67 A steel plate 10 mm thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is 24 mm, the ultimate strength of the steel used is 250 MPa, and the ultimate bonding stress between plate and concrete is 2.1 MPa. Knowing that a factor of safety of 3.60 is desired when $P = 18$ kN, determine (*a*) the required width a of the plate, (*b*) the minimum depth b to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)



(a) Based on tension in the plate.

$$A = (a-d)t$$

$$P_U = \sigma_U A$$

$$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a-d)t}{P}$$

Solving for a,

$$a = d + \frac{(F.S.)P}{\sigma_U t} = 0.024 + \frac{(3.60)(18\times10^3)}{(250\times10^6)(0.010)}$$

$a = 0.04992$ m $\qquad$ $a = 49.9$ mm ◄

(b) Based on shear between plate and concrete slab.

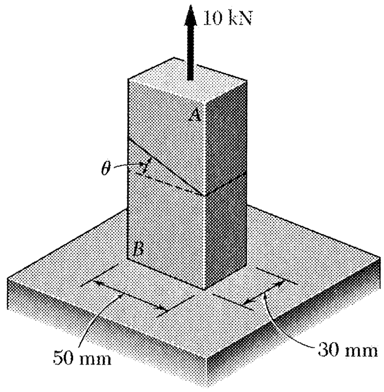
A = perimeter × depth $= (2a + 2t)\,b$ $\qquad$ $\tau_U = 2.1\times10^6$ Pa

$P_U = \tau_U A = 2\tau_U (a+t)\,b$ $\qquad$ $F.S. = \frac{P_U}{P}$

Solving for b, $\qquad$ $b = \frac{(F.S.)P}{2(a+t)\tau_U} = \frac{(3.60)(18\times10^3)}{(2)(0.04992+0.010)(2.1\times10^6)}$

$b = 0.25748$ m $\qquad$ $b = 257$ mm ◄

Problem 1.68



1.68 The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

$$A_o = (0.05)(0.03) = 0.0015\ m^2$$

$$P = 10\ kN \qquad P_u = (F.S)P = 30\ kN$$

Based on tensile stress

$$\sigma_u = \frac{P_u}{A_o}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_u A_o}{P_u} = \frac{(17\times10^6)(0.0015)}{30\times10^3} = 0.85$$

$$\cos\theta = 0.922 \qquad \theta = 22.8^\circ \qquad \theta \geq 22.8^\circ$$

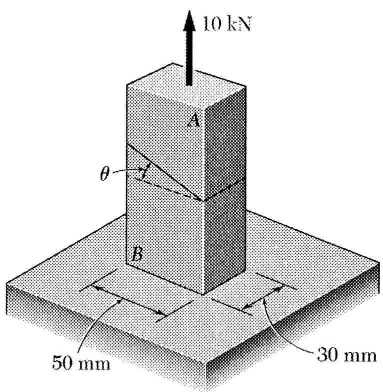
Based on shearing stress $\quad \tau_u = \frac{P_u}{A_o}\sin\theta\cos\theta = \frac{P_u}{2A_o}\sin 2\theta$

$$\sin 2\theta = \frac{2A_o\tau_u}{P_u} = \frac{(2)(0.0015)(9\times10^6)}{30\times10^3} = 0.9$$

$$2\theta = 64.16^\circ \qquad \theta = 32.1^\circ \qquad \theta \leq 32.1^\circ$$

Hence $\quad 22.8^\circ \leq \theta \leq 32.1^\circ$

Problem 1.69



1.69 The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint*: Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

$$A_o = (0.05)(0.03) = 0.0015\ m^2$$

At the optimum angle $\quad (F.S.)_\sigma = (F.S.)_\tau$

Normal stress: $\sigma = \frac{P}{A_o}\cos^2\theta \quad \therefore \quad P_{u,\sigma} = \frac{\sigma_u A_o}{\cos^2\theta}$

$$(F.S.)_\sigma = \frac{P_{u,\sigma}}{P} = \frac{\sigma_u A_o}{P\cos^2\theta}$$

Shearing stress: $\tau = \frac{P}{A_o}\sin\theta\cos\theta \quad \therefore \quad P_{u,\tau} = \frac{\tau_u A_o}{\sin\theta\cos\theta}$

$$(F.S.)_\tau = \frac{P_{u,\tau}}{P} = \frac{\tau_u A_o}{P\sin\theta\cos\theta}$$

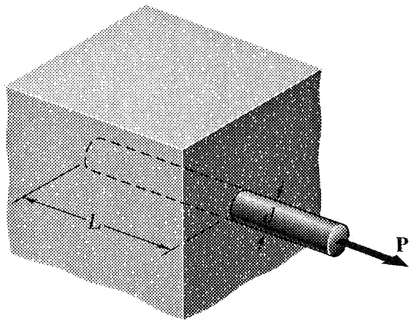
Equating: $\frac{\sigma_u A_o}{P\cos^2\theta} = \frac{\tau_u A_o}{P\sin\theta\cos\theta}$;

Solving: $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\tau_u}{\sigma_u} = \frac{9}{17} = 0.529 \qquad$ (a) $\theta_{opt} = 27.9^\circ$

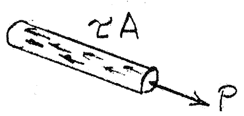
(b) $\quad P_u = \frac{\sigma_u A_o}{\cos^2\theta} = \frac{(17\times10^6)(0.0015)}{\cos^2 27.9^\circ} = 32.65\ kN$

$$F.S. = \frac{P_u}{P} = \frac{32.64}{10} = 3.26$$

Problem 1.70



1.70 A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)



For shear, $A = \pi d L$

$$P = \tau_{all} A = \tau_{all} \pi d \cdot L$$

For tension, $A = \frac{\pi}{4} d^2$

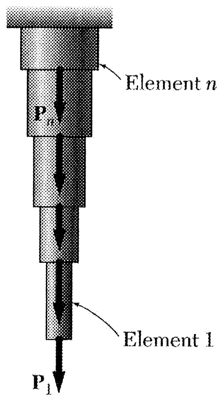
$$P = \sigma_{all} A = \sigma_{all}\left(\frac{\pi}{4} d^2\right)$$

Equating, $\tau_{all} \pi d L = \sigma_{all} \frac{\pi}{4} d^2$

Solving for L,

$$L_{min} = \sigma_{all} d / 4 \tau_{all}$$

PROBLEM 1.C1



1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$, with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program to determine the average stress in each element of the rod. (*b*) Use this program to solve Probs. 1.2 and 1.4.

SOLUTION

FORCE IN ELEMENT i:

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^{i} P_k$$

AVERAGE STRESS IN ELEMENT i:

Area $= A_i = \frac{1}{4}\pi d_i^2$ Ave stress $= \dfrac{F_i}{A_i}$

PROGRAM OUTPUTS

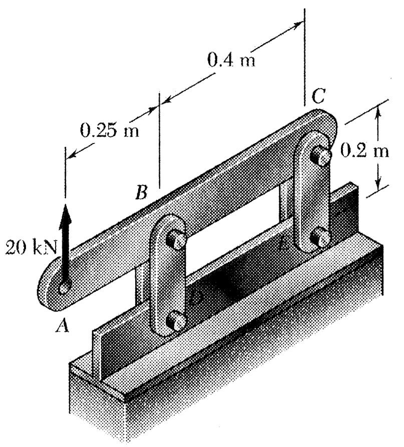
Problem 1.2

Element	Stress (MPa)
1	81.487
2	-18.108

Problem 1.4

Element	Stress (MPa)
1	35.651
2	42.441

PROBLEM 1.C2

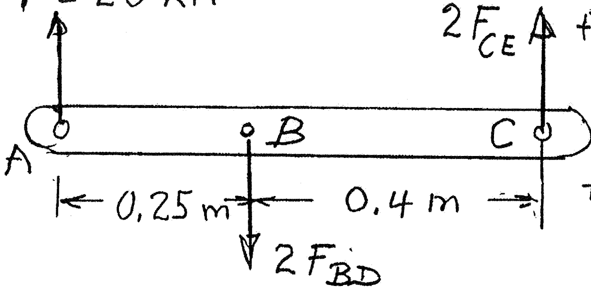


1.C2 A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (2) the average normal stress in the links connecting pins *C* and *E*, (3) the average shearing stress in pin *B*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *B* in member *ABC*, (6) the average bearing stress at *C* in member *ABC*. (*b*) Check your program by comparing the values obtained for *d* = 16 mm with the answers given for Probs. 1.7 and 1.27. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve part *c*, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

FORCES IN LINKS

F.B. DIAGRAM OF ABC:



$+\circlearrowleft \Sigma M_C = 0:\ 2F_{BD}(BC) - P(AC) = 0$

$F_{BD} = P(AC)/2(BC)$ (TENSION)

$+\circlearrowleft \Sigma M_B = 0:\ 2F_{CE}(BC) - P(AB) = 0$

$F_{CE} = P(AB)/2(BC)$ (COMP.)

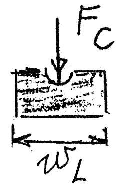
(1) LINK BD

Thickness $= t_L$

$A_{BD} = t_L(w_L - d)$

$\sigma_{BD} = +F_{BD}/A_{BD}$

(2) LINK CE



Thickness $= t_L$

$A_{CE} = t_L w_L$

$\sigma_{CE} = -F_{CE}/A_{CE}$

(3) PIN B

$\tau_B = F_{BD}/(\pi d^2/4)$

(4) PIN C

$\tau_C = F_{CE}/(\pi d^2/4)$

(5) BEARING STRESS AT B

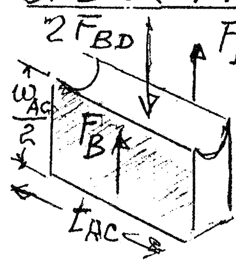
Thickness of member $AC = t_{AC}$

Sig Bear $B = F_{BD}/(d\, t_{AC})$

(6) BEARING STRESS AT C

Sig Bear $C = F_{CE}/(d\, t_{AC})$

SHEARING STRESS IN ABC UNDER PIN B



$F_B = \tau_{AC}\, t_{AC}\,(w_{AC}/2)$

$\Sigma F_y = 0:\ 2F_B = 2F_{BD}$

$\tau_{AC} = \dfrac{2F_{BD}}{t_{AC}\, w_{AC}}$

(CONTINUED)

PROBLEM 1.C2 CONTINUED **PROGRAM OUTPUTS**

INPUT DATA FOR PARTS (a), (b), (c): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 10 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C	
10.00	78.13	-21.70	206.90	79.58	325.00	125.00	
11.00	81.25	-21.70	170.99	65.77	295.45	113.64	
12.00	84.64	-21.70	143.68	55.26	270.83	104.17	
13.00	88.32	-21.70	122.43	47.09	250.00	96.15	
14.00	92.33	-21.70	105.56	40.60	232.14	89.29	
15.00	96.73	-21.70	91.96	35.37	216.67	83.33	
16.00	101.56	-21.70	80.82	31.08	203.12	78.13	← (b)
17.00	106.91	-21.70	71.59	27.54	191.18	73.53	
18.00	112.85	-21.70	63.86	24.56	180.56	69.44	
19.00	119.49	-21.70	57.31	22.04	171.05	65.79	
20.00	126.95	-21.70	51.73	19.89	162.50	62.50	
21.00	135.42	-21.70	46.92	18.04	154.76	59.52	
22.00	145.09	-21.70	42.75	16.44	147.73	56.82	
23.00	156.25	-21.70	39.11	15.04	141.30	54.35	
24.00	169.27	-21.70	35.92	13.82	135.42	52.08	
25.00	184.66	-21.70	33.10	12.73	130.00	50.00	
26.00	203.13	-21.70	30.61	11.77	125.00	48.08	
27.00	225.69	-21.70	28.38	10.92	120.37	46.30	
28.00	253.91	-21.70	26.39	10.15	116.07	44.64	
29.00	290.18	-21.70	24.60	9.46	112.07	43.10	
30.00	338.54	-21.70	22.99	8.84	108.33	41.67	

(c) ANSWER : 16 mm ≤ *d* ≤ 22 mm ← (c)

CHECK: For d = 22 mm, Tau AC = 65 MPa < 90 MPa O.K.

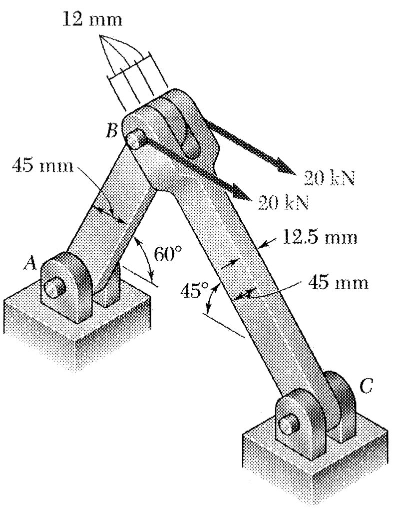
INPUT DATA FOR PART (d): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 8 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	406.25	156.25
11.00	81.25	-21.70	170.99	65.77	369.32	142.05
12.00	84.64	-21.70	143.68	55.26	338.54	130.21
13.00	88.32	-21.70	122.43	47.09	312.50	120.19
14.00	92.33	-21.70	105.56	40.60	290.18	111.61
15.00	96.73	-21.70	91.96	35.37	270.83	104.17
16.00	101.56	-21.70	80.82	31.08	253.91	97.66
17.00	106.91	-21.70	71.59	27.54	238.97	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	156.25	-21.70	39.11	15.04	176.63	67.93
24.00	169.27	-21.70	35.92	13.82	169.27	65.10
25.00	184.66	-21.70	33.10	12.73	162.50	62.50
26.00	203.13	-21.70	30.61	11.77	156.25	60.10
27.00	225.69	-21.70	28.38	10.92	150.46	57.87
28.00	253.91	-21.70	26.39	10.15	145.09	55.80
29.00	290.18	-21.70	24.60	9.46	140.09	53.88
30.00	338.54	-21.70	22.99	8.84	135.42	52.08

(d) ANSWER : 18 mm ≤ *d* ≤ 22 mm ← (d)

CHECK: For d = 22 mm, Tau AC = 81.25 MPa < 90 MPa O.K.

PROBLEM 1.C3

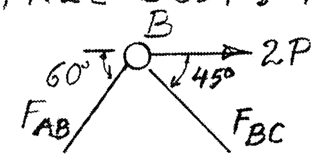


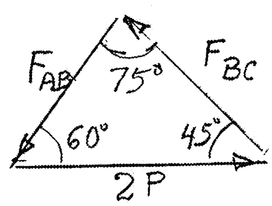
1.C3 Two horizontal 20 kN forces are applied to pin *B* of the assembly shown. Each of the three pins at *A*, *B*, and *C* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 12.5 mm to 37.5 mm, using 1.25-mm increments, (1) the maximum value of the average normal stress in member *AB*, (2) the average normal stress in member *BC*, (3) the average shearing stress in pin *A*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *A* in member *AB*, (6) the average bearing stress at *C* in member *BC*, (7) the average bearing stress at *B* in member *BC*. (*b*) Check your program by comparing the values obtained for $d = 20$ mm with the answers given for Probs. 1.60 and 1.61. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 250 MPa. (*d*) Solve part *c*, assuming that a new design is being investigated in which the thickness and width of the two members are changed, respectively, from 12.5 to 8 mm and from 45 mm to 60 mm.

SOLUTION

FORCES IN MEMBERS AB AND BC

FREE BODY: PIN B





FROM FORCE TRIANGLE:

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{2P}{\sin 75^\circ}$$

$$F_{AB} = 2P(\sin 45^\circ/\sin 75^\circ)$$

$$F_{BC} = 2P(\sin 60^\circ/\sin 75^\circ)$$

(1) MAX. AVE. STRESS IN AB

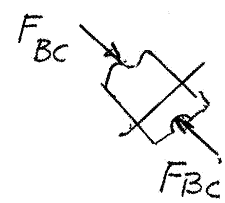
Width $= w$

Thickness $= t$

$$A_{AB} = (w - d)t$$

$$\sigma_{AB} = F_{AB}/A_{AB}$$

(2) AVE. STRESS IN BC



$$A_{BC} = wt$$

$$\sigma_{BC} = F_{BC}/A_{BC}$$

(3) PIN A

$$\tau_A = (F_{AB}/2)/(\pi d^2/4)$$

(4) PIN C

$$\tau_C = (F_{BC}/2)/(\pi d^2/4)$$

(5) BEARING STRESS AT A

Sig Bear A $= F_{AB}/dt$

(6) BEARING STRESS AT C

Sig Bear C $= F_{BC}/dt$

(7) BEARING STRESS AT B IN MEMBER BC

Sig Bear B $= F_{BC}/2dt$

(CONTINUED)

PROBLEM 1.C3 CONTINUED

PROGRAM OUTPUTS

INPUT DATA FOR PARTS (a), (b), (c): P = 20 kN w = 45 mm t = 12 mm

D mm	SIGAB MPa	SIGBC MPa	TAUA MPa	TAUC MPa	SIGBRGA MPa	SIGBRGC MPa	SIGBRGB MPa	
12.5	77.651	-68.687	$\boxed{\cancel{125.537}}$	$\boxed{\cancel{157.420}}$	201.890	247.275	123.641	
17.5	91.773	-68.687	65.578	80.320	144.216	176.622	88.311	
20	100.950	-68.687	50.209	61.490	126.185	154.544	77.272	←(b)
27.5	144.216	-68.687	26.560	32.524	91.773	112.395	56.201	
37.5	$\boxed{\cancel{336.497}}$	-68.687	14.280	17.493	67.302	82.423	41.211	

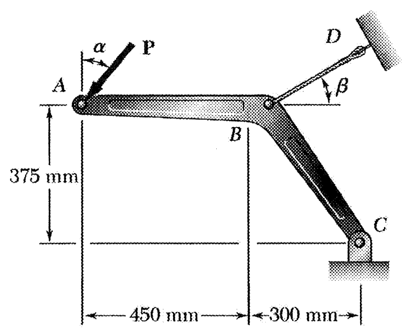
(c) ANSWER: 17.5 mm ≤ d ≤ 27.5 mm

INPUT DATA FOR PART (d): P = 20 kN w = 60 mm t = 8 mm

D mm	SIGAB MPa	SIGBC MPa	TAUA MPa	TAUC MPa	SIGBRA MPa	SIGBRC MPa	SIGBRB MPa
12.5	88.552	-85.857	128.537	157.420	336.497	412.128	206.064
20	105.156	-85.857	50.209	61.490	210.311	257.577	128.792
25	120.180	-85.857	31.131	39.357	168.252	206.064	103.032
31.25	146.305	-85.857	20.568	28.187	134.597	164.853	82.423
37.5	186.944	-85.857	14.280	17.493	112.168	137.376	68.688

(d) ANSWER: 21.25 mm ≤ d ≤ 31.25 mm

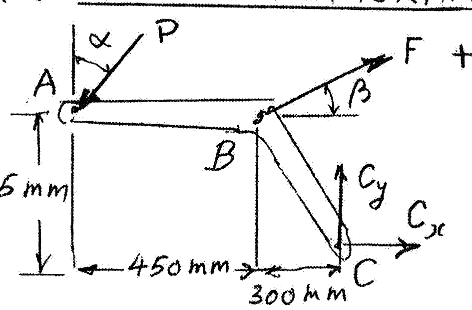
PROBLEM 1.C4



1.C4 A 16 kN force **P** forming an angle α with the vertical is applied as shown to member *ABC*, which is supported by a pin and bracket at *C* and by a cable *BD* forming an angle β with the horizontal. (*a*) Knowing that the ultimate load of the cable is 100 kN, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45°, using increments in α and β corresponding to 0.1 increments in tan α and tan β. (*b*) Check that for any given value of α the maximum value of the factor of safety is obtained for $\beta = 38.66°$ and explain why. (*c*) Determine the smallest possible value of the factor of safety for $\beta = 38.66°$, as well as the corresponding value of α, and explain the result obtained.

SOLUTION

(a) DRAW F. B. DIAGRAM OF ABC:



$$+\circlearrowleft \sum M_C = 0: (P\sin\alpha)(375\text{ mm}) + (P\cos\alpha)(750\text{mm}) - (F\cos\beta)(375\text{ mm}) - (F\sin\beta)(300\text{ mm}) = 0$$

$$F = P\,\frac{15\sin\alpha + 30\cos\alpha}{15\cos\beta + 12\sin\beta}$$

$$F.S. = F_{ult}/F$$

OUTPUT FOR $P = 16$ kN AND $F_{ult} = 100$ kN. ✓

VALUES OF FS

ALPHA \ BETA	0	5.71	11.31	16.70	21.80	26.56	30.96	34.99	38.66	41.99	45.00
0.000	3.125	3.358	3.555	3.712	3.830	3.913	3.966	3.994	4.002	3.995	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.830	3.824	3.807
11.310	2.897	3.113	3.295	3.441	3.551	3.628	3.677	3.703	3.710	3.704	3.687
16.699	2.837	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.438	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.426	3.500	3.547	3.572	3.579	3.573	3.558
30.964	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.356	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.638
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.166	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

↑(b)

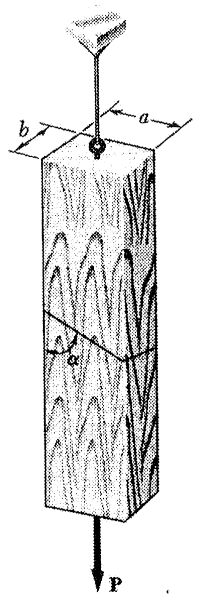
(b) When $\beta = 38.66°$, $\tan\beta = 0.8$ and cable BD is perpendicular to the lever arm BC.

(c) $F.S. = 3.579$ for $\alpha = 26.6°$, P is perpendicular to the lever arm AC

NOTE:

The value $F.S. = 3.579$ is the smallest of the values of F.S. corresponding to $\beta = 38.66°$ and the largest of those corresponding to $\alpha = 26.6°$. The point $\alpha = 26.6°$, $\beta = 38.66°$ is a "saddle point", or "minimax" of the function $F.S.(\alpha,\beta)$.

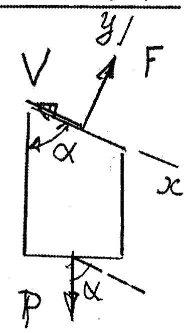
PROBLEM 1.C5



1.C5 A load **P** is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U, respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of a, b, P, σ_U and τ_U, and for values of α from 5 to 85° at 5° intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.31, knowing that σ_U = 1.26 MPa and τ_U = 1.50 MPa for the glue used in Prob. 1.29, and that σ_U = 1.03 MPa and τ_U = 1.47 MPa for the glue used in Prob. 1.31. (*c*) Verify in each of these two cases that the shearing stress is maximum for α = 45°.

SOLUTION

(1) and (2) Draw the F.B. diagram of lower member:



$+\searrow \Sigma F_x = 0: \quad -V + P\cos\alpha = 0 \qquad V = P\cos\alpha$

$+\nearrow \Sigma F_y = 0: \quad F - P\sin\alpha = 0 \qquad F = P\sin\alpha$

Area $= ab/\sin\alpha$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/ab)\sin^2\alpha$$

Shearing stress; $\tau = \dfrac{V}{\text{Area}} = (P/ab)\sin\alpha\cos\alpha$

(3) F.S. for tension (normal stresses)

$FSN = \sigma_U/\sigma$

(4) F.S. for shear:

$FSS = \tau_U/\tau$

(5) OVERALL F.S.:

FS = The <u>smaller</u> of FSN and FSS.

(CONTINUED)

PROBLEM 1.C5 CONTINUED

PROGRAM OUTPUTS

Problem 1.31
a = 150 mm
b = 75 mm
P = 11 kN
SIGU = 1.26 MPa
TAUU = 1.50 MPa

ALPHA	SIG (MPa)	TAU (MPa)	FSN	FSS	FS	
5	.007	.085	169.644	17.669	17.669	
10	.029	.167	42.736	8.971	8.971	
15	.065	.244	19.237	6.136	6.136	
20	.114	.314	11.016	4.773	4.773	
25	.175	.375	7.215	4.005	4.005	
30	.244	.423	5.155	3.543	3.543	
35	.322	.459	3.917	3.265	3.265	
40	.404	.481	3.119	3.116	3.116	
45	.489	.489	2.577	3.068	2.577	◀ (b), (c)
50	.574	.481	2.196	3.116	2.196	
55	.656	.459	1.920	3.265	1.920	
60	.733	.423	1.718	3.543	1.718	
65	.803	.375	1.569	4.005	1.569	
70	.863	.314	1.459	4.773	1.459	
75	.912	.244	1.381	6.136	1.381	
80	.948	.167	1.329	8.971	1.329	
85	.970	.085	1.298	17.669	1.298	

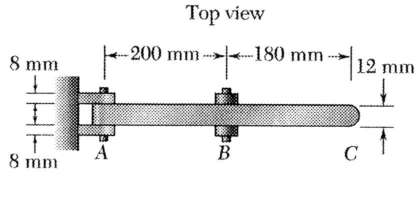
Problem 1.29
a = 125 mm
b = 75 mm
P = 5600 N
SIGU = 1.05 MPa
TAUU = 1.5 MPa

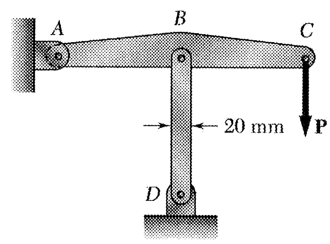
ALPHA	SIG (kPa)	TAU (kPa)	FSN	FSS	FS
5	4.693	56.728	211.574	26.408	26.408
25	116.69	250.243	8.998	5.986	5.986
45	326.669	321.769	3.214	4.586	3.214
60	490.00	282.905	2.143	5.295	2.143
85	648.368	55.877	1.619	26.408	1.619

PROBLEM 1.C6

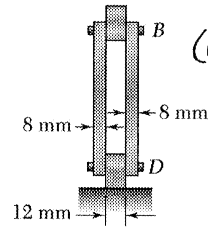
1.C6 Member *ABC* is supported by a pin and bracket at *A* and by two links, which are pin-connected to the member at *B* and to a fixed support at *D*. (*a*) Write a computer program to calculate the allowable load P_{all} for any given values of (1) the diameter d_1 of the pin at *A*, (2) the common diameter d_2 of the pins at *B* and *D*, (3) the ultimate normal stress σ_U in each of the two links, (4) the ultimate shearing stress τ_U in each of the three pins, (5) the desired overall factor of safety *F.S.* Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at *A*, or the shearing stress in the pins at *B* and *D*. (*b* and *c*) Check your program by using the data of Probs. 1.55 and 1.56, respectively, and comparing the answers obtained for P_{all} with those given in the text. (*d*) Use your program to determine the allowable load P_{all}, as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MP for aluminum links, $\tau_U = 100$ MPa for steel pins, and *F.S.* = 3.2.

Top view





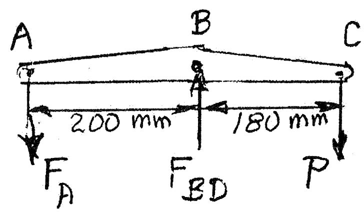
Front view



Side view

SOLUTION

(a) F.B. DIAGRAM OF ABC:



$\Sigma M_A = 0$: $P = \frac{200}{380} F_{BD}$

$\Sigma M_B = 0$: $P = \frac{200}{180} F_A$

(1) For given d_1 of pin A: $F_A = 2(\tau_U/FS)(\pi d_1^2/4)$, $P_1 = \frac{200}{180} F_A$

(2) For given d_2 of pins B and D: $F_{BD} = 2(\tau_U/FS)(\pi d_2^2/4)$, $P_2 = \frac{200}{380} F_{BD}$

(3) For ultimate stress in links BD: $F_{BD} = 2(\sigma_U/FS)(0.02)(0.008)$, $P_3 = \frac{200}{380} F_{BD}$

(4) For ult. shearing stress in pins: P_4 is the smaller of P_1 and P_2

(5) For desired overall F.S.: P_5 is the smaller of P_3 and P_4

If $P_3 < P_4$, stress is critical in links

If $P_4 < P_3$ and $P_1 < P_2$, stress is critical in pin A

If $P_4 < P_3$ and $P_2 < P_1$, stress is critical in pins B and D

PROGRAM OUTPUTS

(b) Prob. 1.53. DATA: $d_1 = 8$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, F.S. = 3.0

$P_{all} = 3.72$ kN. Stress in pin A is critical ◀

(c) Prob. 1.54 DATA: $d_1 = 10$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, F.S. = 3.0

$P_{all} = 3.97$ kN. Stress in pins B and D is critical ◀

(d) DATA: $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa, $\tau_U = 100$ MPa, F.S. = 3.2

$P_{all} = 5.79$ kN. Stress in links is critical ◀